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Citation for published version:

Zwicky, R 2008, 'Unparticles at heavy flavour scales: CP violating phenomena', *Physical Review D*, vol. 77, no. 3, 036004, pp. 1-16. <https://doi.org/10.1103/PhysRevD.77.036004>

Digital Object Identifier (DOI):

[10.1103/PhysRevD.77.036004](https://doi.org/10.1103/PhysRevD.77.036004)

Link:

[Link to publication record in Edinburgh Research Explorer](#)

Document Version:

Early version, also known as pre-print

Published In:

Physical Review D

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Unparticles at heavy flavour scales: CP violating phenomena

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Version of February 3, 2008

Abstract:

Coupling the scale invariant unparticle sector to flavour physics and assuming that it remains scale invariant we investigate its consequences in heavy flavour physics. A drastic feature of unparticle physics is an unusual phase leading to novel CP violating phenomena. We consider the CP asymmetry in the leptonic decay $B^+ \rightarrow \tau^+ \nu$ and the hadronic decay $B_d \rightarrow D^+ D^-$, taking into account constraints of branching ratios and time dependent CP asymmetries. Generic plots are shown and it turns out that there exist parameters for which the CP violation is maximal. A prediction of a large CP asymmetry in $B_d \rightarrow D^+ D^-$ is difficult to achieve in other models without contradicting the current data in other channels. The prediction of a CP asymmetry in leptonic decays, such as $B^+ \rightarrow \tau^+ \nu$, is novel. We identify the CP compensating mode due to the unparticles and show explicitly that it exactly cancels the CP asymmetry of $B^+ \rightarrow \tau^+ \nu$ as demanded by CPT invariance. Building up on earlier works we investigate the breaking of scale invariance, due to the coupling to the Higgs and the size of the effects in the weak sector resorting to a dimensional analysis. An enhancement is observed on the grounds of the relevance of the unparticle interaction operator as compared to the weak four-Fermi term.

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1 Introduction

The possibility of a non-trivial scale invariant sector, weakly coupled to the Standard Model (SM), was advocated by Georgi in [1]. A scale invariant theory does not contain degrees of freedom with isolated masses¹, unlike field theories used in phenomenological particle physics. Georgi called the degrees of freedom of such a theory "unparticles".

A non-trivial scale invariant sector, i.e. non-vanishing coupling, exhibits power-like scaling unlike the logarithmic scaling of QCD at the perturbative or trivial ultraviolet fixed point. The power-like scaling in Minkowski space seems to lead to curious phenomena. For example, the phase space of an unparticle with scaling dimension d_U , which consists of the classical plus anomalous dimension, looks like a number of d_U (possibly non-integer) massless particles [1]. This could lead to interesting signals of missing energy. In a second paper [3] Georgi has pointed out that the unparticle propagator has an unusual phase $e^{-id_U\pi}/\sin(d_U\pi)$ leading to spectacular interference patterns.

By parametrizing a variety of interactions, unparticle phenomena were investigated at various energy scales and domains of particle physics such as electroweak physics [4], [5], [7], collider physics [8], [9], [10] (the latter investigates the (pseudo)resonance structure due to unparticles), DIS [11], [12] B , D -physics [13], [14] [15] [16] [17], light flavour physics [18], [19] $g_\mu - 2$ [13], [20] lepton flavour violation [21],[22], invisible decays [23], cosmology [24] long-range interaction [25] and gravity [26]; All studies are based on the assumption that the theory remains scale invariant until the respective energy domain.

Papers addressing questions of interpretation and the range of scale invariance have appeared. In an illuminating paper by Stephanov [27] the continuous spectrum of the unparticle fields is discretized allowing for interpretation in terms of the language of particle physics. The authors of reference [5] address the question of the range of scale invariance. In the case where the unparticle couples to the Higgs vacuum expectation value (VEV) the latter will render the theory non scale invariant. This raises the question whether unparticle effects are observable in low energy experiments. A follow-up paper has appeared [6], where it is observed that higher dimensional operators can be parametrically enhanced under certain conditions on the scaling dimensions. Moreover this paper contains many physical applications and considers LEP-results to set bounds on the effective suppression scale.

At first sight it seems rather difficult to pursue an analysis in low energy physics. The unparticle effects are parametrized in terms of an effective field theory where no principle is (yet) known to constrain the coefficients and a coupling to the Higgs VEV would take the theory away from scale invariance. On the other hand the novel phases could give rise to such striking phenomena that an investigation seems worthwhile. Moreover we have adapted the analysis of Ref. [5] to the weak sector and find that effects are possible if the unparticle field couples weakly to the Higgs VEV. The size of this coupling is not dictated by any principle and we may therefore regard its smallness as a working assumption.

¹In the absence of a mass scale the only possible particle candidates seem to be massless fields, but these have been shown to be free fields [2].

In the specific model or parametrization used, the unparticle will play the role of the W -boson or charged Higgs in flavour-changing decays. It is well known that (time independent) CP violation manifests itself if there are at least two amplitudes with different relative strong (CP-even) and weak (CP-odd) phases. The phase of the unparticle propagator is CP-even and if we therefore allow for a different weak phase in the unparticle sector the door is opened to novel CP violating phenomena. Decays with one dominant weak amplitude seem particularly suitable, since they do not exhibit sizable CP violation. Moreover the unparticle should propagate at large heavy flavour energies because of the breaking of scale invariance.

We analyze leptonic decays of the type $B \rightarrow \tau \nu$, where the SM and Beyond the Standard Models (BSM) do *not* predict a CP asymmetry and $B_d \rightarrow D^+ D^-$, which is further motivated by the unexpectedly large CP asymmetry measured by the Belle collaboration [43]. We shall also investigate how large the impact of unparticles can be without conflicting with branching ratio and indirect CP asymmetry predictions.

The paper is organized as follows. In section 2 the scenario of the model is described including our parametrization of the effective Lagrangian and some general notation for CP violation is introduced. In section 3 the leptonic decay $B \rightarrow \tau \nu$ and $B_d \rightarrow D^+ D^-$ are investigated followed by a discussion of similar channels. In section 4 we verify a constraint on CP-violation from CPT-invariance; namely that the partial sum of particle and antiparticle rates, with final states rescattering into each other, are equal. In section 5 we present the dimensional analysis of [5] adapted to a weak process. The paper ends with a summary and conclusions in section 6.

In this paper we shall adopt $\Lambda_U = 1 \text{ TeV}$ as the scale of the IR fixed point. It is not difficult to rescale the results to a different scale, in the relevant places ($\Lambda_U/1 \text{ TeV}$) will be shown explicitly in the formulae.

2 Scenario

According to [1], with slightly adapted notations from [5], we shall imagine that at a very high energy scale $M_U \gg 1 \text{ TeV}$ the particle world is described by the standard model fields and a self interacting ultraviolet sector. These two sectors interact with each other via heavy particles of mass M_U . The ultraviolet sector is supposed to contain a non-trivial infrared (IR) fixed point. An example mentioned in [1] is the Banks-Zaks [28], perturbative type, fixed-point. Other examples are gauge theories with fermions in higher dimensional representations which exhibit near conformality, also known as walking. The phase diagram for an arbitrary number of flavours and colours was given in [29] and preliminary lattice studies seem to confirm the theoretical expectations [30]. Below the scale M_U the theory may be described by non-renormalizable interactions

$$\mathcal{L}^{\text{eff}} \sim \frac{1}{M_U^{d_{UV} + (d_{\text{SM}} - 4)}} O_{\text{SM}} O_{UV} \quad , \quad (1)$$

analogous to the four-Fermi interactions connecting the lepton and quark families within the SM. The ultraviolet theory flows into the IR fixed point around some scale $\Lambda_U \sim 1$ TeV which will lead to new degrees of freedom called unparticles described by operators $O_{\text{IR}} \equiv O_U$. Below the scale Λ_U the theory may be described by an effective field theory in terms of the new degrees of freedom

$$\mathcal{L}^{\text{eff}} \sim \frac{\lambda}{\Lambda_U^{d_U + (d_{\text{SM}} - 4)}} O_{\text{SM}} O_U \quad (2)$$

with coupling $\lambda = c_U (\Lambda_U/M_U)^{d_{UV} + (d_{\text{SM}} - 4)}$ and matching coefficient c_U . Since we do not have a concrete theory at hand c_U will be a free parameter to be constrained by experimental data.

To make use of Lagrangians of the form (2) it will prove sufficient to know the coupling of an unparticle degree of freedom with given momentum P to its field operator for calculating decays into unparticles. Moreover via the optical theorem or dispersion relation it is possible to obtain the propagator and study interference effects, at the tree level, of unparticles and SM particles. In this paper we are concentrating on the latter effect.

The propagator may be defined from its dispersion representation

$$\Delta_U(P^2) \equiv i \int_0^\infty d^4x e^{ip \cdot x} \langle 0 | T O_U(x) O_U^\dagger(0) | 0 \rangle = \int_0^\infty \frac{ds}{\pi} \frac{\text{Im}[\Delta_U(s)]}{s - P^2 - i0} + \text{s.t.} \quad (3)$$

where we have assumed that the unparticle state $|P\rangle$ satisfies $P^2 \geq 0$ and $P_0 > 0$. The abbreviation s.t. stands for possible subtraction terms associated to a possible non convergence in the ultraviolet. The imaginary part is given by

$$\text{Im}[\Delta_U(P^2)] = |\langle 0 | O_U(0) | P \rangle|^2 = A_{d_U} (P^2)^{d_U - 2}. \quad (4)$$

The P^2 dependence is solely determined by the scaling property of the operator O_U . The factor A_{d_U} is a priori an arbitrary normalization constant which has been chosen to be

$$A_{d_U} = 16\pi^{5/2} \frac{\Gamma(d_U + 1/2)}{\Gamma(d_U - 1)\Gamma(2d_U)} \quad (5)$$

in reference [1]. It is the phase space volume of d_U massless particles. This choice was motivated by the fact that (4) exhibits the same functional behaviour as d_U massless particles. This together with the fact that unparticles would (presumably) escape particle detectors has led Georgi to point out [1]: "Unparticle stuff with scale dimension d_U looks like a non-integral number d_U of invisible particles." With (4) the dispersive integral (3) is elementary:

$$\Delta_U(P^2) = \frac{A_{d_U}}{2 \sin(d_U \pi)} \frac{1}{(-P^2 - i0)^{2 - d_U}} \xrightarrow{P^2 \geq 0} \frac{A_{d_U}}{2 \sin(d_U \pi)} \frac{e^{-id_U \pi}}{(P^2)^{2 - d_U}}. \quad (6)$$

We observe that the unusual phase, due to analytic continuation to Minkowski space, is due to the non-integral scaling dimension d_U . It has been shown in [3] that the discontinuity of the propagator yields the imaginary part (4), which we have already implicitly assumed in the dispersion representation (3). The phase is accompanied by a term

$\sin(d_{\mathcal{U}}\pi)$ in the denominator. In simple cases of direct CP asymmetries this term will cancel and in more complicated cases it will partly cancel. The cancelation of this term will play a crucial role when we verify the equality of partial rates of particles and antiparticles, which is a consequence of CPT, in section 4. Plots of the function $A_{d_{\mathcal{U}}}$ and $A_{d_{\mathcal{U}}}/\sin(d_{\mathcal{U}}\pi)$ can be found in the appendix A.

A strong (CP-even) phase in a propagator appears spectacular and is not an element of common models in particle physics. The phase factor is due to dynamics in the unparticle or scale invariant sector. The exactly solvable two-dimensional Thirring model, which contains fermions with a current-current interaction term, is an example where the dynamical phase can be seen explicitly, c.f. appendix B.2. The anomalous dimension is a function of the coupling constant and assumes the free field value in the case where the coupling is taken to zero. We would like to add that in the Thirring model, due to the fermion selection rule, the anomalous phase is not so immediately observable. It is also interesting to note that there is a connection between the non-trivial phase and the causality structure. The commutator of the unparticle field in the vacuum is, c.f. appendix B.1 Eq. (A.6),

$$\langle 0|[O_{\mathcal{U}}(x), O_{\mathcal{U}}(0)]|0\rangle = -i \operatorname{sign}(x_0)\theta(x^2)(x^2)^{-d_{\mathcal{U}}}f(d_{\mathcal{U}}), \quad (7)$$

where $f(d_{\mathcal{U}})$ is a function explicitly given in (A.6). The commutator vanishes for space-like $x^2 < 0$ separation and obeys causality. For generic $d_{\mathcal{U}}$ the support is inside the light cone which seems in agreement with the spectrum $P^2 \geq 0$. For an integer value, $d_{\mathcal{U}} = n$, the commutator behaves as $\sim \delta^{(n-1)}(x^2)$ and has support on the light cone only. Note that in the latter case there is no CP-even phase in the unparticle propagator.

We shall now discuss the possible values of $d_{\mathcal{U}}$. In the upper range $d_{\mathcal{U}} = 2$ is singled out, since we observe that for $d_{\mathcal{U}} \leq 2$ no subtraction terms are needed. Moreover the singularity of $d_{\mathcal{U}}$ approaching an integer value larger than 2 has been interpreted in [3] as the a $d_{\mathcal{U}}$ -particle cut which should not be attempted to be described by a single unparticle field. In the lower range the value $d_{\mathcal{U}} = 1$ seems special since it corresponds to the free massless field,

$$\lim_{d_{\mathcal{U}} \rightarrow 1} \Delta_{\mathcal{U}}(P^2) = \frac{1}{P^2}. \quad (8)$$

It is also observed that for $d_{\mathcal{U}} \leq 1$ the dispersion integral does not converge in the infrared. This might be interpreted by the fact that the field decreases even slower than the free massless field in coordinate space. It has been shown that for $d_{\mathcal{U}} < 1$ the conformal group does not admit unitary representations [32]. Moreover in reference [1] it was noted that the decay into an unparticle has a non-integrable singularity in the decay rate for $d_{\mathcal{U}} < 1$. We shall therefore think of $d_{\mathcal{U}}$ as being

$$1 < d_{\mathcal{U}} < 2 \quad (9)$$

or parametrising with respect to the free field limit, $d_{\mathcal{U}} = 1 + \gamma$ with anomalous dimension $0 < \gamma < 1$ at the non-trivial fixed point.

2.1 Parametrization of the effective Lagrangian

In this section we shall give our parametrisation of the coupling of the unparticle sector to the SM. We will investigate charged-flavour decays and therefore it is sufficient to give the couplings to that sector. We couple a vectorial unparticle operator O_U to a scalar and a pseudoscalar density. The unparticle will therefore be a charged Lorentz-scalar and play the role of a charged Higgs rather than a W -boson. We parametrize the effective Lagrangian as follows²

$$\begin{aligned} \mathcal{L}^{\text{eff}} = & \frac{\lambda_S^{q'q}}{\Lambda_U^{d_U-1}} (\bar{q}' q) O_U + \frac{\lambda_S^{\nu l}}{\Lambda_U^{d_U-1}} (\bar{\nu} l) O_U \\ & + \frac{\lambda_P^{q'q}}{\Lambda_U^{d_U-1}} (\bar{q}' \gamma_5 q) O_U + \frac{\lambda_P^{\nu l}}{\Lambda_U^{d_U-1}} (\bar{\nu} \gamma_5 l) O_U + \text{h.c.}, \end{aligned} \quad (10)$$

where $q' = (u, c, t)$, $q = (d, s, b)$, $\nu = (\nu_e, \nu_\mu, \nu_\tau)$ and $l = (e, \mu, \tau)$ are summations over the families. In the notation of Eq. (2), $d_{\text{SM}} = 3$. The weak (CP-odd) phases are parametrized as deviation from the phases of the CKM matrix $V_{q'q}$ and analogously the leptons as deviations from the PMNS matrix $U_{\nu l}$.

$$\lambda_S^{q'q} = e^{i\phi_{q'q}^S} |\lambda_S^{q'q}| \quad \phi_{q'q}^S = \arg[V_{q'q}] + \delta\phi_{q'q}^S \quad . \quad (11)$$

The Lagrangian is a non vectorial copy of the charged current sector in the SM. This allows us to apply up to some level the same tools in the unparticle sector as in the SM. Note that the unparticle carries charge, unlike the Lagrangian used in [1].

We would like to stress that the Lagrangian in Eq.(10), in the absence of an explicit model realising the unparticle scenario, is not dictated by any structure and is therefore only an example. Other Dirac structures with the same flavour transitions are possible. The axial and vector structures, for example, can be coupled to a transversal unparticle O_U^μ ($\partial_\mu O_U^\mu = 0$) or to a derivative coupling of a scalar unparticle $\partial^\mu O_U$

$$\delta\mathcal{L}^{\text{eff}} = \frac{\lambda_{\partial(V,A)}^{q'q}}{\Lambda_U^{d_U}} (\bar{q}' \gamma_\mu (\gamma_5) q) \partial^\mu O_U + \dots \quad (12)$$

The former leads to a propagator transversal propagator $\sim (-g_{\mu\nu} + p_\mu p_\nu / p^2)$ which vanishes by transversality when coupled to a pseudoscalar particle of momentum p_μ , which is the case in the examples considered in this paper. The latter leads to an identical contribution as the scalar and pseudoscalar contribution in the examples considered, $\lambda_{S,P} \leftrightarrow \text{const} \cdot \lambda_{\partial(V,A)}$, where $\lambda_{\partial(V,A)}$ is parametrically suppressed by one power of Λ_U as compared to $\lambda_{S,P}$. We shall give the results in terms of $\lambda_{S,P}$ in the paper but also indicate explicitly how they change for a $\lambda_{\partial(V,A)}$ -coupling. Please note that although we have

²The channel $B^+ \rightarrow \tau^+ \nu$ is mediated by a $(P \times S) + (P \times P)$ structure whereas $B_d \rightarrow D^+ D^-$ decays via a $(S \times P)$ interaction. The vector and axial couplings are discussed in the text above and we do not consider tensor couplings since they do not couple to single scalar particles.

just stated that $\lambda_{S,P}$ and the $\lambda_{\partial(V,A)}$ are equivalent in the examples considered, their role in model building, in regard to $SU(2)_L$ for example, might be rather different since the former couples fermions of opposite chirality whereas the latter couples fermions of the same chirality.

2.2 General formulae for branching ratios and CP asymmetries

In this section we shall give the formulae for the branching ratio and CP asymmetries, used later on, in the case of two amplitudes with different strong and weak phases. This paragraph is completely general in principle, but we shall have in mind that one amplitude is due the SM and the other is due to the unparticles. For a decay $\bar{B} \rightarrow CD$ we parametrize

$$\bar{\mathcal{A}}(\bar{B} \rightarrow X) = A_1 e^{i\delta_1} e^{i\phi_1} + A_2 e^{i\delta_2} e^{i\phi_2} \quad , \quad (13)$$

where the δ_i denote the CP-even phases and ϕ_i the CP-odd phases. The branching ratio \mathcal{B} and the CP averaged branching ratios are given by

$$\mathcal{B} = \mathcal{B}^0 f_\Delta \quad \bar{\mathcal{B}} = \mathcal{B}^0 \bar{f}_\Delta \quad , \quad (14)$$

where

$$\begin{aligned} \mathcal{B}^0 &= \tau(\bar{B}) \frac{1}{16\pi m_B^3} \lambda^{1/2}(m_B^2, m_C^2, m_D^2) |A_1|^2 \quad \Delta = \frac{|A_2|^2}{|A_1|^2} \\ f_\Delta &= (1 + 2\Delta \cos(\phi_{12} + \delta_{12}) + \Delta^2) \\ \bar{f}_\Delta &= (1 + 2\Delta \cos(\phi_{12}) \cos(\delta_{12}) + \Delta^2) \quad , \end{aligned} \quad (15)$$

and $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$, $\phi_{12} = \phi_1 - \phi_2$ and $\delta_{12} = \delta_1 - \delta_2$. In the case where the transitions

$$B \rightarrow CD \leftarrow \bar{B} \quad (16)$$

are possible, the B -meson is neutral and the dynamical mixing of B_d and \bar{B}_d leads to a time dependence in the CP asymmetry. In the case where the coefficients q and p , relating the flavour and mass eigenstates of the neutral system, assume $|q/p| = 1$, the lifetime difference $\Delta\Gamma/\Delta M \ll 1$ the CP asymmetry assumes the following form

$$\mathcal{A}_{\text{CP}}(B_d \rightarrow CD) \equiv \frac{\Gamma[\bar{B} \rightarrow CD] - \Gamma[B \rightarrow \bar{C}\bar{D}]}{\Gamma[\bar{B} \rightarrow CD] + \Gamma[B \rightarrow \bar{C}\bar{D}]} = S_{CD} \sin(\Delta Mt) - C_{CD} \cos(\Delta Mt) \quad . \quad (17)$$

Both assumptions mentioned above are satisfied for the B_d system. The sign convention is such that the $b \rightarrow cd$ decay rate enters with a plus sign, please note $\bar{B} \equiv \bar{B}_0 \equiv \bar{B}_d \sim (b\bar{d})$ [35]. Writing $q/p = e^{-i\phi_d}$, where $\phi_d = 2\beta$ is the mixing phase of the B_d system, $\lambda = q/p(\mathcal{A}/\bar{\mathcal{A}})$, then the (time independent) CP asymmetry assumes the following form

$$C = \frac{1 - |\lambda|^2}{1 + |\lambda|^2} = \frac{2\Delta}{\bar{f}_\Delta} \sin(\delta_{12}) \sin(\phi_{12}) \quad . \quad (18)$$

In the case where the system CD is a CP eigenstate with eigenvalue $\xi_{CD} = \pm 1$, which is a particular realization of (16), the time dependent CP asymmetry assumes the following form

$$S = \xi_{CD} \frac{2\text{Im}(\lambda)}{1 + |\lambda|^2} = \xi_{CD} \frac{-1}{f_\Delta} (\sin(\phi_d + 2\phi_1) + 2\Delta \cos(\delta_{12}) \sin(\phi_d + \phi_{12}) + \Delta^2 \sin(\phi_d + 2\phi_2)). \quad (19)$$

For $B_d \rightarrow J/\Psi K_s$ $\xi_{J/\Psi K_s} = -1$, $\phi_1 \simeq 0$ and there is no sizable second amplitude in the SM and therefore $\Delta \simeq 0$ and $\phi_2 \rightarrow 0$ and the gold plated formula $S_{J/\Psi K_s} = \sin(2\beta)$ is recovered.

3 A leptonic and a hadronic decay

3.1 $B^+ \rightarrow \tau^+ \nu$; scale invariant sector at 5 GeV

In the standard model charged pseudoscalars decaying to a lepton and a neutrino are of particular interest because of their simple dependence on the pseudoscalar decay constant and the CKM matrix element, see below.

The novel feature when adding unparticles is a CP asymmetry. We will investigate how large this asymmetry can be, remaining consistent with the the branching ratio measurement

Below we will give the decay amplitude for $B^+ \rightarrow \tau^+ \nu$ for the SM and the unparticle contribution with effective Lagrangian as given in (10).

The unparticle is propagating at the scale m_B and we therefore assume that the scale invariant sector extends down to the m_B scale. The SM and unparticle graphs are shown in Fig. 1, where we have indicated the phase.

The additional unparticle amplitude leads to a slight complication. As a matter of fact in experiment we do not observe the neutrino flavour but an inclusive measurement on the neutrino flavour is performed since the neutrinos are not detectable. In the case where there is only one amplitude, as in the SM, unitarity of the PMNS matrix hides this fact from the final formula. This is not the case for the unparticle amplitude and we shall therefore derive formulae for $B^+ \rightarrow \tau^+ \nu \equiv \sum_l B^+ \rightarrow \tau^+ \nu_l$ via $B^+ \rightarrow \tau^+ \nu_l$. The amplitude is the sum of two incoherent terms of opposite parity in the final state³

$$\mathcal{A}(B^+ \rightarrow \tau^+ \nu_l) = \frac{G_F}{\sqrt{2}} V_{ub}^* U_{\tau \nu_l} f_B m_\tau \cdot \left([\bar{\nu} \tau] (1 + \Delta_{\tau \nu_l}^S e^{-id_u \pi} e^{-i\phi_l^S}) + [\bar{\nu} \gamma_5 \tau] (-1 + \Delta_{\tau \nu_l}^P e^{-id_u \pi} e^{-i\phi_l^P}) \right), \quad (20)$$

³The amplitude (20) displays the famous helicity suppression in the SM due to its chiral structure which manifests itself in the fact that the amplitude is proportional to the lepton mass. For a pseudoscalar coupling, as in the charged Higgs model, or the one used here, the helicity suppression is relieved as can be inferred from Eq. (21). If we were to use a derivative coupling $\partial_\mu O$ to the axial vector (12) instead of a pseudoscalar coupling as in (10) then the following substitution, $m_B^2/(m_b m_\tau) \rightarrow m_B^2/\Lambda_U^2$, in Eq. (21) would reproduce the result for the derivative coupling.

where $\phi_l^D = \delta\phi_{\text{ub}}^P - \delta\phi_{\tau\nu_l}^D$ for $D = (S, P)$, $l = (e, \mu, \tau)$. The B -meson decay constant is defined as $m_b \langle 0 | \bar{b} i \gamma_5 u | B^+ \rangle = f_B m_B^2$, where we neglect isospin breaking effects. The ratio of unparticle to SM amplitude is

$$\Delta_{\tau\nu_l}^D \equiv \frac{|\lambda_D^{\tau\nu_l}|}{|U_{\tau\nu_l}|} \tilde{\Delta}_{\tau\nu} \equiv r_l^D \tilde{\Delta}_{\tau\nu},$$

$$\tilde{\Delta}_{\tau\nu} = \frac{|\lambda_P^{\text{ub}}|}{|V_{\text{ub}}|} \frac{A_{d_{\mathcal{U}}}}{2 \sin(d_{\mathcal{U}}\pi)} \frac{m_B^2}{m_b m_\tau} \frac{(G_F/\sqrt{2})^{-1}}{m_B^2} \left(\frac{m_B^2}{\Lambda_{\mathcal{U}}^2} \right)^{d_{\mathcal{U}}-1}. \quad (21)$$

We will now make a simplifying assumption in order to simplify the analysis. We impose the left-handed chirality on the unparticle sector i.e. $\lambda_S^{\tau\nu_l} = -\lambda_P^{\tau\nu_l}$ and $(\Delta_{\tau\nu} \equiv \Delta_{\tau\nu}^{(S,P)}, \delta\phi_{\tau\nu} \equiv \delta\phi_{\tau\nu}^S, r_l \equiv r_l^{(S,P)})$. This means that the amplitudes for opposite parity give the same result and this allows us to combine the two amplitudes into one. The branching fractions to a specific neutrino flavour final state are

$$\mathcal{B}(B^+ \rightarrow \tau^+ \nu_l) = \mathcal{B}_{\tau\nu}^{\text{SM}} |U_{\tau\nu_l}|^2 f_{\Delta_{\tau\nu_l}} \quad \bar{\mathcal{B}}(B^+ \rightarrow \tau^+ \nu_l) = \mathcal{B}_{\tau\nu_l}^{\text{SM}} |U_{\tau\nu_l}|^2 \bar{f}_{\Delta_{\tau\nu_l}}, \quad (22)$$

with f and \bar{f} as in (15), $\phi_{12} = -\phi_l$, $\delta_{12} = -d_{\mathcal{U}}\pi$. The familiar SM branching fraction reads

$$\mathcal{B}_{\tau\nu}^{\text{SM}} = \tau(B^+) \frac{G_F^2}{8\pi} |V_{\text{ub}}|^2 f_B^2 m_B m_\tau^2 \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \quad (23)$$

and does not depend on the neutrino flavour. Please note that in the SM $\mathcal{B}_{\tau\nu}^{\text{SM}} = \bar{\mathcal{B}}_{\tau\nu}^{\text{SM}}$. The experimentally tractable or neutrino inclusive branching fraction is

$$\begin{aligned} \mathcal{B}(B^+ \rightarrow \tau^+ \nu) &= \sum_l \mathcal{B}(B^+ \rightarrow \tau^+ \nu_l) \\ &= \mathcal{B}_{\tau\nu}^{\text{SM}} \sum_l |U_{\tau\nu_l}|^2 (1 + 2r_l \tilde{\Delta}_{\tau\nu} \cos(\phi_l + d_{\mathcal{U}}\pi) + (r_l \tilde{\Delta}_{\tau\nu})^2) \\ &= \mathcal{B}_{\tau\nu}^{\text{SM}} (1 + \sum_l |U_{\tau\nu_l}|^2 (2r_l \tilde{\Delta}_{\tau\nu} \cos(\phi_l + d_{\mathcal{U}}\pi) + (r_l \tilde{\Delta}_{\tau\nu})^2)) \quad . \quad (24) \end{aligned}$$

The formula could be further simplified if the r_l were independent of l , which we shall assume shortly below. The CP averaged branching fraction is

$$\bar{\mathcal{B}}(B^+ \rightarrow \tau^+ \nu) \equiv \mathcal{B}_{\tau\nu}^{\text{SM}} \mathcal{F} = \mathcal{B}_{\tau\nu}^{\text{SM}} (1 + \sum_l |U_{\tau\nu_l}|^2 (2r_l \tilde{\Delta}_{\tau\nu} \cos(\phi_l) \cos(d_{\mathcal{U}}\pi) + (r_l \tilde{\Delta}_{\tau\nu})^2)) \quad (25)$$

The CP asymmetry assumes the following form

$$\mathcal{A}_{\text{CP}}(\tau\nu) \equiv \frac{\Gamma(B^- \rightarrow \tau^- \bar{\nu}) - \Gamma(B^+ \rightarrow \tau^+ \nu)}{\Gamma(B^- \rightarrow \tau^- \bar{\nu}) + \Gamma(B^+ \rightarrow \tau^+ \nu)} = \frac{2\tilde{\Delta}_{\tau\nu}}{\mathcal{F}} \sin(d_{\mathcal{U}}\pi) \sum_l \sin(\phi_l) r_l |U_{\tau\nu_l}|^2, \quad (26)$$

where \mathcal{F} is implicitly defined in Eq. (25). Let us note that the CP violation encountered here is proportional to $\sim \text{Im}[V_{\text{ub}}^* \lambda_P^{\text{ub}} U_{\tau\nu} \lambda_S^{\tau\nu*}]$, which is hidden in the formula above,



Figure 1: (left) SM diagram for $B \rightarrow \tau \nu$ (right) unparticle diagram with CP odd phase $e^{id_U\pi}$. The unparticle is denoted by a double line.

and is the product of two quadratic reparametrization invariants. The effect is entirely proportional to the sine of the phase difference between the CKM (PMNS) and the unparticle flavour sector and can therefore not occur in the SM. In order to do a qualitative assessment we shall study the case where there is no flavour dependent perturbation in the neutrino sector and therefore drop the label l . The formulae for the CP averaged branching ratio and the CP asymmetry then simplify to

$$\bar{\mathcal{B}}(B^+ \rightarrow \tau^+ \nu) \rightarrow \mathcal{B}_{\tau\nu}^{\text{SM}}(1 + 2\Delta_{\tau\nu} \cos(\phi) \cos(d_U\pi) + \Delta_{\tau\nu}^2) \xrightarrow{\phi=\pm\pi/2} \mathcal{B}_{\tau\nu}^{\text{SM}}(1 + \Delta_{\tau\nu}^2),$$

$$\mathcal{A}_{\text{CP}}(\tau\nu) \rightarrow \frac{2\Delta_{\tau\nu} \sin(\phi) \sin(d_U\pi)}{1 + 2\Delta_{\tau\nu} \cos(\phi) \cos(d_U\pi) + \Delta_{\tau\nu}^2} \xrightarrow{\phi=\pm\pi/2} \frac{\pm 2|\Delta_{\tau\nu}| |\sin(d_U\pi)|}{1 + \Delta_{\tau\nu}^2}, \quad (27)$$

where in the last step we have simplified the formulae further by setting the weak phase difference to $90(270)^\circ$ ⁴. N.B. in the notation used in Eq. (26) $\mathcal{A}_{\text{CP}}(\tau\nu) = -C_{\tau\nu}$. This choice maximizes the CP violation for appropriate values for $\Delta_{\nu l}$. Before we are able to constrain the CP violation with the rate we have to give the theoretical and experimental results of the latter.

The following hadronic parameters, $\tau^{B^+} = 1.643 \text{ ps}$, $f_B = (189 \pm 27) \text{ MeV}$ a lattice average from [36] and $|V_{ub}| = 3.64(24) \cdot 10^{-3}$ from the fit to the angles of the CKM triangle [36], are used to estimate the SM branching fraction

$$\mathcal{B}(B^+ \rightarrow \tau^+ \nu)_{\text{theory}}^{\text{SM}} = 83(40) \cdot 10^{-6}. \quad (28)$$

We have doubled the uncertainty due to $|V_{ub}|$. This estimate has to be compared with the measurements at the B -factories

units 10^{-6}	$\bar{\mathcal{B}}(B^+ \rightarrow \tau^+ \nu)$
BaBar[37](223M BB)	90(60)(10)
Belle[38](449M BB)	179(53)(48)
HFAG[34]	132(49) .

(29)

⁴N.B. $\sin(d_U\pi) < 0$ for $1 < d_U < 2$ as assumed throughout this paper (9). This is the reason for the absolute values in the equation above.

Weak phase $\phi = 90(270)^\circ$, flavour independent perturbation neutrino sector

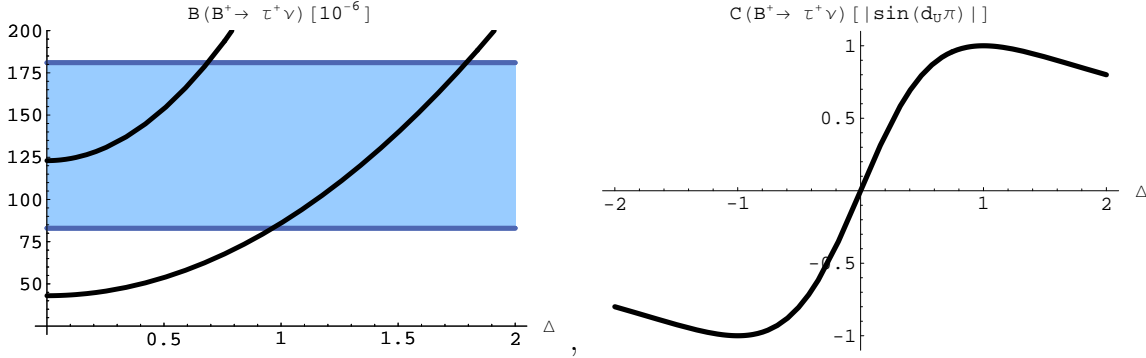


Figure 2: A weak phase difference $\phi = 90(270)^\circ$ is assumed here for $\Delta_{\nu\mu}$ positive(negative). (left) Branching fraction (27) as a function of $\Delta_{\tau\nu}$. The black bands correspond to the SM estimate (28) at $\Delta_{\tau\nu} = 0$. The blue band corresponds to the HFAG bounds in Eq. (29). (right) The CP asymmetry as a function of $\Delta_{\tau\nu}$ in units of $|\sin(d_U\pi)|$. The scale $\Lambda_U = 1 \text{ TeV}$ is chosen here. N.B. in the notation used in Eq. (26) $\mathcal{A}_{\text{CP}}(\tau\nu) = -C_{\tau\nu}$

In Fig. 2 (left) the branching fraction (27) is plotted as a function of $\Delta_{\tau\nu}$ with uncertainty taken from the SM estimate (28) at $\Delta_{\tau\nu} = 0$. The blue band corresponds to the HFAG bounds in Eq. (29). The CP asymmetry is plotted to the right of that figure. The branching ratio does not set limits on the amount of CP violation, demanding the uncertainty bands to be tangent at worst $|\Delta_{\tau\nu}| < 1.8$. Even in the case where the HFAG and theory uncertainty are halved, the value $|\Delta_{\tau\nu}| = 1$, at which the CP asymmetry is maximal, is still consistent.

Weak phase $\phi \neq 90(270)^\circ$, flavour independent perturbation neutrino sector

In this subsection we shall repeat the analysis for a general weak phase difference and show two dimensional plots in the variables (ϕ, d_U) for different ratios of effective couplings. The quantity $\Delta_{\tau\nu}$ (21), used in the previous paragraph, depends on the ratio of effective coupling and scaling dimension as follows

$$\begin{aligned} \Delta_{\tau\nu} &= \rho_{\tau\nu} \frac{A_{d_U}}{2 \sin(d_U \pi)} \frac{m_B^2}{m_b m_\tau} \frac{(G_F/\sqrt{2})^{-1}}{m_B^2} \left(\frac{m_B^2}{\Lambda_U^2} \right)^{d_U-1} \\ &\simeq 2300 \left(2.8 \cdot 10^{-5} \frac{\Lambda_U}{1 \text{ TeV}} \right)^{d_U-1} \frac{A_{d_U}}{\sin(d_U \pi)} \rho_{\tau\nu}, \end{aligned}$$

where

$$\rho_{\tau\nu} \equiv \frac{|\lambda_P^{\text{ub}} \lambda_{(S,P)}^{\tau\nu}|}{|V_{\text{ub}} U_{\tau\nu}|}. \quad (30)$$

A plot relating $\Delta_{\tau\nu}$ and d_U can be found in appendix A, Fig. 9. In Fig. 3 (right) the CP asymmetry $C_{\tau\nu}$ is plotted as a function of (ϕ, d_U) for $\rho_{\tau\nu} = (10^0, 10^{-2}, 10^{-4})$. The pattern is clearly regular and the condition for a large asymmetry is $|\Delta_{\tau\nu}| \sim 1$.

For smaller values of $\rho_{\tau\nu}$ the amount of possible CP violation is decreasing because the condition mentioned above cannot be satisfied. The constraint on the branching fraction, Fig. 3 (left), is defined by the following acceptance function

$$A(d_{\mathcal{U}}, \phi, \rho) = (1 - r(d_{\mathcal{U}}, \phi, \rho))\Theta(1 - r(d_{\mathcal{U}}, \phi, \rho)),$$

$$r(d_{\mathcal{U}}, \phi, \rho) = \frac{1}{\Delta B} |\mathcal{B}_{\tau\nu}^{\text{SM}}(1 + 2\Delta_{\tau\nu} \cos(\phi) \cos(d_{\mathcal{U}}\pi) + \Delta_{\tau\nu}^2) - \mathcal{B}^{\text{HFAG}}| \quad (31)$$

for $\mathcal{B}_{\tau\nu}^{\text{SM}} = 83 \cdot 10^{-6}$, $\mathcal{B}^{\text{HFAG}} = 132 \cdot 10^{-6}$ and for the quantity ΔB we add the uncertainty of the SM prediction and the HFAG value linearly to $\Delta B \simeq 80 \cdot 10^{-6}$. This function assumes values between 0 and 1, where 1 signifies maximal agreement and 0 means that the point is excluded; or in other words we consider predictions with a deviation larger than ΔB as excluded.

For smaller values of $\rho_{\tau\nu}$ the linear term for the branching ratio in Eq. (27) becomes dominant and a regular pattern in $\cos(\phi)$ emerges. Note that since the predicted branching fraction is lower than the central value from experiment, the weak angle $\phi = 180^\circ$ is currently disfavoured since it would lower the theory prediction even more.

3.2 Discussion and remarks on $B \rightarrow \mu\nu$, $D \rightarrow \mu\nu$, $B_s \rightarrow \mu^+\mu^-$ etc

We have seen that applying the unparticle scenario to the leptonic decay $B \rightarrow \tau\nu$ leads to CP violation. There is no experimental data available that gives both the negative and positive charged semileptonic decay rates, i.e. quotes (bounds) on CP asymmetry in a semileptonic decay.

The current data on $B \rightarrow \tau\nu$ do not allow us to set bounds on the amount of possible CP violation. The amount of events at BaBar and Belle are of the order ~ 20 . An improvement in theory, in particular on the B -meson decay constant, and the large statistics of a Super B -factory would of course improve the situation. Unfortunately the decay $B^+ \rightarrow \tau^+\nu$ will not be possible or competitive at LHCb because of the neutrino final state and the intricacies in the τ detection, whether $D(D_s) \rightarrow (\tau, \mu)\nu$ decays are possible at LHCb is currently under investigation.

We shall comment on other leptonic modes. They are all described by the same formula (23) for $B \rightarrow \tau\nu$ with obvious substitutions for V_{ub} , f_B , m_B and m_τ . We may also consider the D -decays assuming that the scale invariant sector extends to ~ 2 GeV. The decay $D^+ \rightarrow \mu^+\nu$ is measured by CLEO [39] the ~ 50 events lead to a thirteen percent accuracy. The decay constant $f_D^+ = 220(20)$ MeV is taken as an average value of theory determinations from the table in [39] and $|V_{cd}| = 0.227$ [35]. The Cabibbo allowed decays $D_s^+ \rightarrow \mu^+\nu$ are measured as well [35], although with less precision. The decay constant $f_{D_s}^+ = 264(36)$ is obtained from an average of $f_{D_s}^+/f_D^+ = 1.20(5)$ of the table in [39] and $|V_{cs}| = 0.957(17)(93)$ [35]. A summary of the experimental [35] and theory

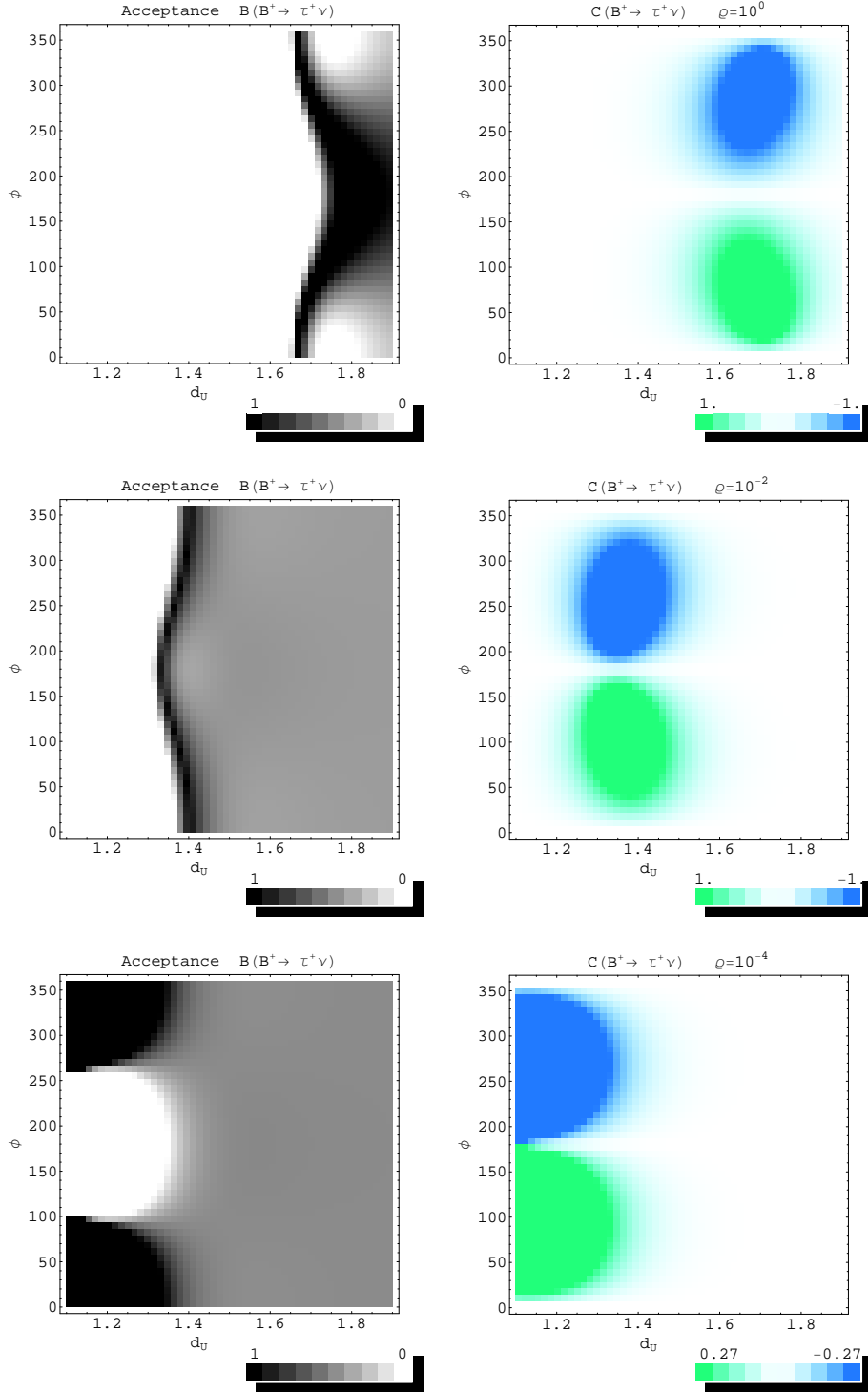


Figure 3: A horizontal line of figures corresponds to different fractions of effective couplings $\rho = 10^{0,-2,-4}$ as defined in (30). (left) Constraints on the (ϕ, d_U) parameter-space from the branching fraction. The values in the dark regions are allowed whereas white ones are excluded, c.f. Eq. 31 for a more details. (right) The CP asymmetry as a function of (ϕ, d_U) . The scale $\Lambda_U = 1 \text{ TeV}$ is chosen here.

predictions is:

	$B \rightarrow \tau\nu$	$B \rightarrow \mu\nu$	$B \rightarrow e\nu$
Experiment	$132(49) \cdot 10^{-6}$	$< 17 \cdot 10^{-7}$	$< 9.8 \cdot 10^{-6}$
Theory	$83(50\%) \cdot 10^{-6}$	$3.7(50\%) \cdot 10^{-7}$	$8.4(50\%) \cdot 10^{-12}$
	$D \rightarrow \tau\nu$	$D \rightarrow \mu\nu$	$D \rightarrow e\nu$
Experiment	$< 2.1 \cdot 10^{-3}$	$4.4(7) \cdot 10^{-4}$	$< 2.4 \cdot 10^{-5}$
Theory	$1.1(20\%) \cdot 10^{-3}$	$4.3(20\%) \cdot 10^{-4}$	$1.0(20\%) \cdot 10^{-10}$
	$D_s \rightarrow \tau\nu$	$D_s \rightarrow \mu\nu$	$D_s \rightarrow e\nu$
Experiment	$6.4(15) \cdot 10^{-2}$	$6.3(18) \cdot 10^{-3}$	not available
Theory	$5.5(30\%) \cdot 10^{-2}$	$5.7(30\%) \cdot 10^{-3}$	$1.3(30\%) \cdot 10^{-7}$,

The B decays are predicted to 50% due to uncertainties in f_B and $|V_{ub}|$, whereas the $D(D_s)$ decays have a lower uncertainty 20(30)% due to $f_D(f_{D_s})$. The helicity suppression in the SM is apparent from the table.

Repeating the analysis for $D^+ \rightarrow \mu^+\nu$, as shown in Fig. 2, we obtain that $|\Delta_{D \rightarrow \mu\nu}| < 0.65$ which still allows for a rather large CP asymmetry, $|C_{D \rightarrow \mu\nu}| < 0.9$.

The prediction of these modes in the SM is solid and a significant deviation would be a clear hint for new physics. In particular one expects larger rates in models where the helicity suppression is relieved. An example is the charged Higgs or the effective Lagrangian used in this paper. The charged Higgs does not predict a significant CP asymmetry whereas in unparticle models it is possible and therefore a CP asymmetry could be used to discriminate between the models.

We would also like to mention the decay $K^+ \rightarrow \mu^+\nu$, the KLOE collaboration reports ~ 860 events and a branching ratio $\mathcal{B}(K^+ \rightarrow \mu^+\nu(\gamma)) = 0.6366(9)(15)$ [40]. On the one hand it seems unreasonable that the scale invariant sector could extend to ~ 500 MeV but on the other this channel has the largest statistics. If we assume that theory predicts the rate to 5%(10%) this would roughly bound $|\Delta_{K \rightarrow \mu\nu}| < 20(30)$ and the CP asymmetry to $|C_{K \rightarrow \mu\nu}| < 0.4(0.55)$.

Finally a comment about $B_{(d,s)} \rightarrow \mu^+\mu^-$. This channel is rare since it is a flavour-changing neutral decay further suppressed by the coupling of the Z and the helicity of final states, $\mathcal{B}(B_{(d,s)} \rightarrow \mu^+\mu^-)^{\text{SM}} \sim 10^{-10}(10^{-8})$. The branching ratio is not yet measured, the bounds are about one and half order of magnitude away from the SM prediction. An analysis along the lines of $B \rightarrow \tau\nu$ does not make sense since there are no direct constraints in that channel. A possibility would be to combine it with constraints from $\Delta M_{(d,s)}$, which are measured, as advocated in reference [14].

3.3 $B_d \rightarrow D^+ D^-$; scale invariant sector at 2 GeV

The decay $B_d \rightarrow D^+ D^-$ corresponds to a $b \rightarrow \bar{c} d$ transition at the quark level and is colour allowed. It has the same quark level transition as $B_d \rightarrow J/\Psi \pi_0$ but two complications arise as compared to the latter. First, since it is colour allowed it receives sizable contributions from a gluonic penguin[41] and second the final states combine into a sum of isospin $I = 0$ and $I = 1$ waves which have in general different final state interaction phases. Ultimately we will neglect the penguins in our analysis, to be discussed below. Our motivation to investigate the $B_d \rightarrow D^+ D^-$ is driven by the measurement of a large CP asymmetry by the Belle collaboration [43]⁵. The SM expectation is $C_{D^+ D^-}^{\text{SM}} \simeq -0.05$.

	$C_{D^+ D^-}$	$S_{D^+ D^-}$
BaBar[42](364M BB)	0.11(22)(07)	-0.54(34)(06)
Belle[43](535M BB)	-0.91(23)(06)	-1.13(37)(09)
HFAG	-0.37(17)	-0.75(26)

(32)

It has to be said that the Belle result is somewhat moderated by a significantly lower value from BaBar [42] with opposite sign. Note that the central values from Belle also violate the general bound $C^2 + S^2 \leq 1$.

It shall be our goal to see how large a CP asymmetry $C_{D^+ D^-}$ the unparticles scenario can generate and still be consistent with the branching fraction and the time dependent CP asymmetry.

In our analysis the unparticle will replace the W in the tree level amplitude in, c.f. Fig. 4 (left). We therefore assume that the scale invariant sector extends to the D -meson scale ~ 2 GeV.

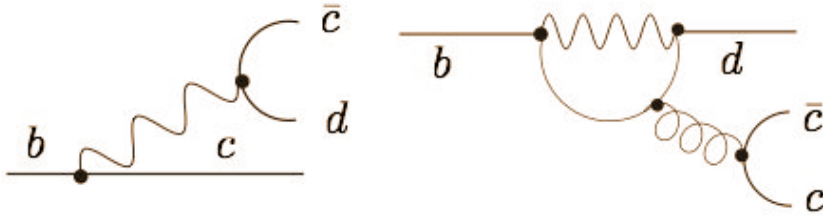


Figure 4: $b \rightarrow d\bar{c}$ (left) tree diagram, (right) penguin diagram

We shall first reconsider the situation in the SM before we move on to the unparticles. Writing the amplitude as the sum of the tree and penguin topology

$$\mathcal{A}(B_d \rightarrow D^+ D^-) = \mathcal{A}_T + \mathcal{A}_P = \mathcal{A}_T(1 - e^{i\delta_{\text{PT}}} e^{i\gamma} r_{\text{PT}}), \quad (33)$$

⁵I am grateful to Christopher Smith for drawing my attention to this measurement.

the ratio of penguin to tree amplitude r_{PT} can then be estimated by the Bander-Silverman-Soni mechanism [41], c.f. [44] or [45] for an updated analysis,

$$\Delta_{\text{PT}} \simeq 0.08 \quad \delta_{\text{PT}} \simeq 205^\circ \quad . \quad (34)$$

This allows us to obtain the asymmetries from (18) and (19),

$$C_{D^+D^-}^{\text{SM}} \simeq -0.05 \quad S_{D^+D^-}^{\text{SM}} \simeq -0.78 \quad . \quad (35)$$

Comparing with the experimental results (32) we infer that the SM is in good agreement with the time dependent CP asymmetry $S_{D^+D^-}$. The direct CP asymmetry $C_{D^+D^-} \simeq 0.05$ is about two standard deviations lower than the HFAG value 0.37(17). In view of the non consistency of the two measurements it is certainly wise to wait for updates from the B -factories. We will in the following neglect the penguin contribution in regard to its moderate size (34) in the SM. We will also neglect the "unparticle penguin". The ratio of the unparticle penguin amplitude to the unparticle amplitude is expected to be of the same size as in the SM, unless the up-type transition is enhanced by the effective couplings. We are therefore implicitly assuming that $|\lambda_{(S,P)}^{\text{ub}}\lambda_{(S,P)}^{\text{ud}}| \lesssim |\lambda_{(S,P)}^{\text{cb}}\lambda_{(S,P)}^{\text{cd}}|$.

We will describe the amplitude $B_d \rightarrow D^+D^-$ within the naive factorization approximation. Naive factorization describes colour allowed modes (topology as in Fig. 4 to the left) like $B \rightarrow \pi^+\pi^+$ and $B_d \rightarrow D^+\pi^-$ with at least one fast or light meson with an accuracy of around 10 – 20% level. For $B_d \rightarrow D^+D^-$, factorization in general and naive factorization are not expected to hold. The overlap of the emitted D^+ -meson with the $B_d \rightarrow D^-$ transition is expected to be relatively large. However it is empirically observed that naive factorization still works reasonably well. We shall account for final state interactions, not included in naive factorisation, by an isospin analysis which is presented in the appendix C. The effect is that the amplitude receives a contribution $\cos((\delta_1 - \delta_0)/2) \simeq \pm 0.63(15)$, c.f. (A.17). In fact the sign is not determined but since it enters in the square in the observables it is of no concern here. The amplitude for $B_d \rightarrow D^+D^-$ in the SM is

$$\begin{aligned} \mathcal{A}(B_d \rightarrow D^+D^-) &= \frac{G_F}{\sqrt{2}} V_{\text{cb}}^* V_{\text{cd}} a_1 f_D ((m_B^2 - m_D^2) f_+^{BD}(m_D^2) + m_D^2 f_-^{BD}(m_D^2)) \\ &\times \cos((\delta_1 - \delta_0)/2) e^{i(\delta_1 - \delta_0)/2} \equiv \mathcal{A}_{\text{DD}}^{\text{SM}}, \end{aligned} \quad (36)$$

where $a_1 = C_2 + C_1/3 \simeq 1$ is the colour allowed combination of tree level Wilson coefficients and the D -meson decay constant is defined as $m_c \langle 0 | \bar{c} i \gamma_5 d | D^- \rangle = f_D m_D^2$, where we neglect effects due to isospin breaking. The $B \rightarrow D$ form factor can be parametrized by use of Lorentz covariance as

$$\langle D | \bar{b} \gamma_\mu c | B \rangle = f_+^{BD}(q^2) (p_B + p_D)_\mu + f_-^{BD}(q^2) q_\mu, \quad (37)$$

with momentum transfer $q = p_B - p_D$. The form factors are related to the famous Isgur-Wise function $f_+^{BD}(q^2) = \sqrt{\frac{m_B + m_D}{4m_B m_D}} \xi(w)$, $f_-^{BD}(q^2) = -\sqrt{\frac{m_B - m_D}{4m_B m_D}} \xi(w)$ in the heavy quark limit. Here $w = v \cdot v' = (m_B^2 + m_D^2 - q^2)/(2m_B m_D)$. Whereas the normalization of

the Isgur-Wise function $\xi(1) = 1$ follows from charge normalization in the heavy quark limit the values around maximum recoil are much less known. We shall take the value $f_+^{BD}(0) = 0.54$ from [47] and scale it up to $q^2 = m_D^2$ by use of a single pole model [46], $\xi(w) \sim \sqrt{2/(w+1)}(w_{\max} - w(m_{B_c^*}^2)/(w - w(m_{B_c^*}^2))$. The B_c^* -meson has the correct quantum numbers $J^P = 1^+$ and its mass is the same in the heavy quark limit as $m_{B_c} = 6.29 \text{ GeV}$ [35]. We obtain $f_+(m_D^2) \simeq 0.7$. With $f_D = 220 \text{ MeV}$. we get

$$\bar{\mathcal{B}}(B_d \rightarrow D^+ D^-)_{\text{theory}}^{\text{SM}} = 1.7(10) \cdot 10^{-4} \quad (38)$$

as a theory estimate, where the bulk of the uncertainty quoted is due to the isospin final state interaction phases (A.17). This estimate has to be compared to the experimental value [35]

$$\bar{\mathcal{B}}(B_d \rightarrow D^+ D^-)_{\text{PDG}} = 1.9(6) \cdot 10^{-4} \quad . \quad (39)$$

The agreement seems accidentally good in regard to the approximations made.

As in the previous section we parametrize the amplitude

$$\mathcal{A}(B_d \rightarrow D^+ D^-) \equiv \mathcal{A}_{\text{DD}}^{\text{SM}}(1 + \Delta_{\text{DD}} e^{-i\phi_{\mathcal{U}}} e^{-i\phi}) \quad (40)$$

with $\mathcal{A}_{\text{DD}}^{\text{SM}}$ as given in (36) and relative weak phase $\phi \equiv \delta\phi_{\text{cb}} - \delta\phi_{\text{cd}}$. The ratio of SM to unparticle amplitude is⁶

$$\Delta_{\text{DD}} = \frac{|\lambda_S^{\text{cb}} \lambda_P^{\text{cd}}|}{|V_{\text{cb}} U_{\text{cd}}|} \frac{1}{a_1} \frac{A_{d_{\mathcal{U}}}}{2 \sin(d_{\mathcal{U}} \pi)} \frac{m_D^2}{m_c(m_b - m_c)} \frac{(G_F/\sqrt{2})^{-1}}{m_D^2} \left(\frac{m_D^2}{\Lambda_{\mathcal{U}}^2} \right)^{d_{\mathcal{U}}-1}. \quad (41)$$

Note that, unlike for $B \rightarrow \tau \nu$, the negative parity of the D -meson selects only the λ_P^{cd} coupling in the final vertex. The observables are obtained from Eq. (18) and (19) with $\xi_{D^+ D^-} = 1$, $\phi_d = 2\beta$, $\phi_1 = 0$, $\phi_2 = -\phi$ and $\delta_{12} = d_{\mathcal{U}} \pi$:

$$\begin{aligned} \mathcal{B}_{\text{DD}} &= \mathcal{B}_{\text{DD}}^{\text{SM}} f_{\Delta_{\text{DD}}} & \bar{\mathcal{B}}_{\text{DD}} &= \mathcal{B}_{\text{DD}}^{\text{SM}} \bar{f}_{\Delta_{\text{DD}}}, \\ C_{\text{DD}} &= \frac{2\Delta_{\text{DD}}}{\bar{f}_{\Delta_{\text{DD}}}} \sin[\phi] \sin[d_{\mathcal{U}} \pi], \\ S_{\text{DD}} &= \frac{-1}{\bar{f}_{\Delta_{\text{DD}}}} (\sin[2\beta] + 2\Delta_{\text{DD}} \cos[d_{\mathcal{U}} \pi] \sin[2\beta - \phi] + \Delta_{\text{DD}}^2 \sin[2\beta - 2\phi]) \end{aligned} \quad (42)$$

and

$$\mathcal{B}_{\text{DD}}^{\text{SM}} = \tau(B_d) \frac{G_F^2}{32\pi m_B} a_1^2 f_D^2 ((m_B^2 - m_D^2) f_+^{BD}(m_D^2) + m_D^2 f_-^{BD}(m_D^2))^2 |V_{\text{cb}}^* V_{\text{cd}}|^2. \quad (43)$$

Weak phase $\phi = 90(270)^\circ$

In order to look for maximal CP violation we may again set the weak phase difference to $90(270)^\circ$ in the formulae in Eq. (42). In Fig. 5 (left) the branching fraction is plotted

⁶A derivative coupling $\partial_\mu O_{\mathcal{U}}$ to the vector Dirac-structure (12) instead of a scalar coupling as in (10) would lead to a change of $m_D^2/(m_c(m_b - m_c)) \rightarrow m_D^2/\Lambda_{\mathcal{U}}^2$ in Eq. (41).

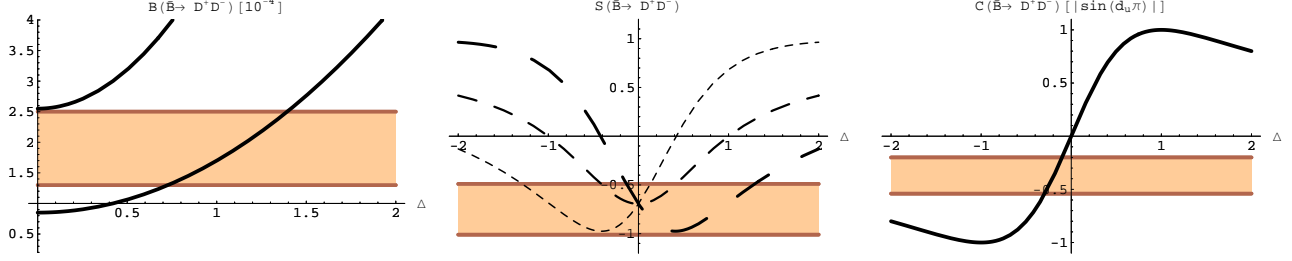


Figure 5: A weak phase difference $\phi = 90(270)^\circ$ is assumed here for Δ_{DD} positive (negative). (left) Branching fraction (42) as a function of Δ_{DD} . The black bands correspond to the SM estimate (38) at $\Delta_{DD} = 0$. The brown-red band corresponds to the HFAG bounds in Eq. (39). (middle) Time dependent CP asymmetry $S_{D^+D^-}$ as a function of Δ_{DD} for $d_U = 1.1, 1.5, 1.9$ where the dashes get shorter for larger values of d_U . The interpolation between those values is fairly smooth. (right) The CP asymmetry as a function of Δ_{DD} in units of $|\sin(d_U\pi)|$.

as a function of Δ_{DD} with uncertainty taken from the SM estimate (38) at $\Delta_{DD} = 0$. The brown-red band corresponds to the HFAG bounds in Eq. (39). The new feature as compared to the $B \rightarrow \tau\nu$ analysis is the constraint from $S_{D^+D^-}$ which corresponds to the figure in the middle. The CP asymmetry is plotted to the right of that figure. Once more the branching ratio does not set limits on the amounts of CP violation, in fact the uncertainties are very similar as in $B \rightarrow \tau\nu$. Demanding the uncertainty bands to be tangential at worst results in $|\Delta_{DD}| < 1.5$. The constraints from $S_{D^+D^-}$ do depend on the scaling dimension. The parameter $d_U = 1.1$ for example seems slightly disfavoured as compared to the value $d_U = 1.9$

Weak phase $\phi \neq 90(270)^\circ$

We investigate the two dimensional parameter space (ϕ, d_U) for different ratios of effective couplings. These quantities relate to Δ_{DD} (41) as follows

$$\begin{aligned} \Delta_{DD} &= \rho_{DD} \frac{A_{d_U}}{2 \sin(d_U \pi)} \frac{m_D^2}{m_c(m_b - m_c)} \frac{(G_F/\sqrt{2})^{-1}}{m_D^2} \left(\frac{m_D^2}{\Lambda_U^2} \right)^{d_U-1} \\ &\simeq 17 \cdot 10^3 \left(3.5 \cdot 10^{-6} \frac{\Lambda_U}{1 \text{ TeV}} \right)^{d_U-1} \frac{A_{d_U}}{\sin(d_U \pi)} \rho_{DD}, \end{aligned}$$

where

$$\rho_{DD} \equiv \frac{|\lambda_S^{\text{cb}} \lambda_P^{\text{cd}}|}{|V_{\text{cb}} U_{\text{cd}}|}. \quad (44)$$

A plot relating Δ_{DD} and d_U can be found in appendix A, Fig. 9. In Fig. 6 (right) CP asymmetry $C_{D^+D^-}$ is plotted as a function (ϕ, d_U) for $\rho_{DD} = (10^0, -2, -4)$. The pattern is very similar in its form to $B \rightarrow \tau\nu$. A large asymmetry is obtained for $|\Delta_{\tau\nu}| \sim 1$, which cannot be attained for smaller values ρ_{DD} . The constraint on the branching fraction, Fig. 3 (left), and the CP asymmetry $S_{D^+D^-}$ are evaluated with the same kind of acceptance function as for $B \rightarrow \tau\nu$ (31). The corresponding values for the CP asymmetry

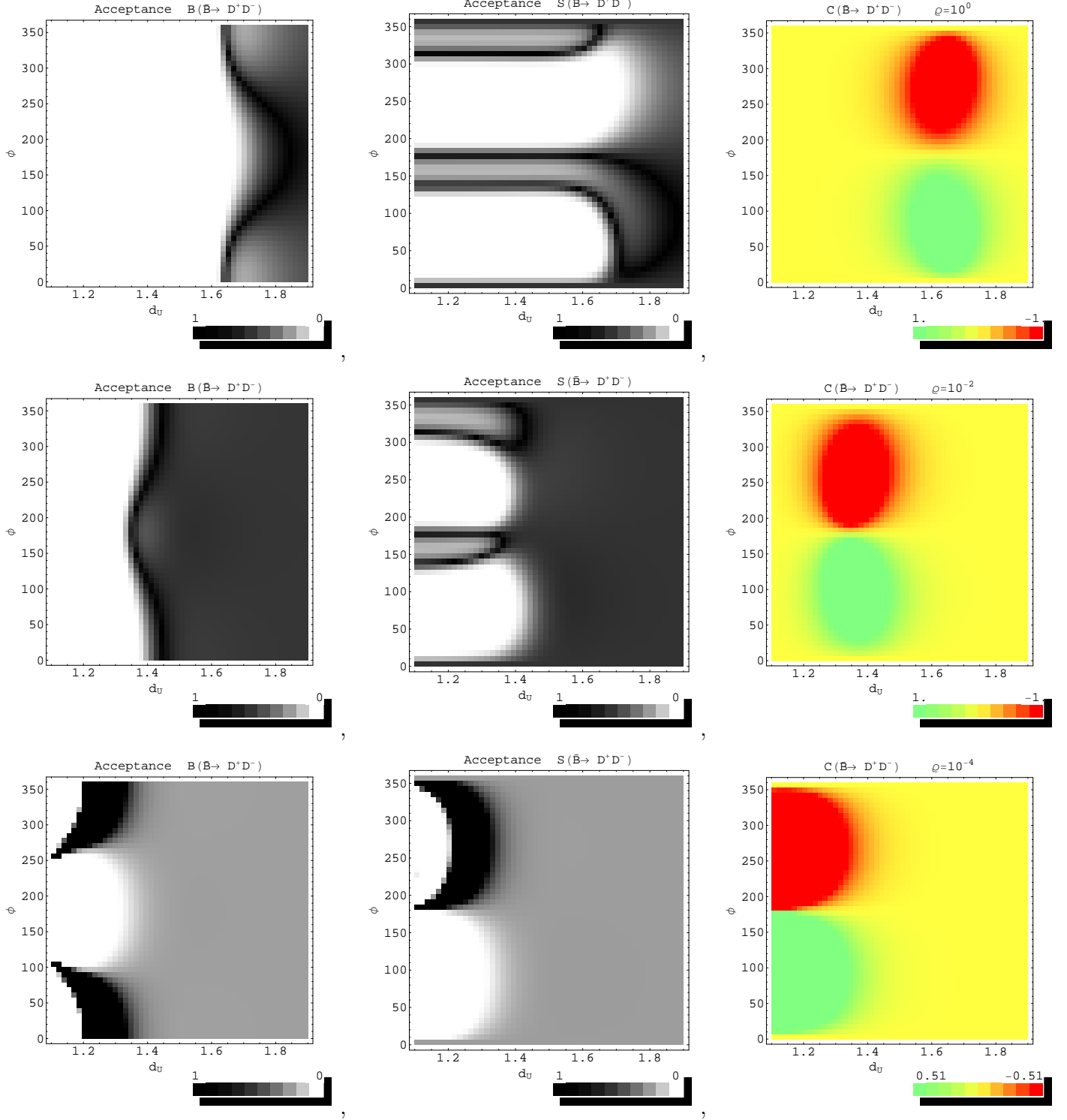


Figure 6: The observables with fractions of effective couplings $\rho = 10^{0,-2,-4}$, as defined in (44), are plotted from the top of the figure to the bottom. Constraints on the (ϕ, d_U) parameter-space from (left) the branching fraction and (middle) the CP asymmetry $S_{D^+D^-}$ (middle). The values in the dark regions are allowed whereas white ones are disfavoured. c.f. text for more details. (right) The CP asymmetry $C_{D^+D^-}$ as a function of (ϕ, d_U) . The scale $\Lambda_U = 1 \text{ TeV}$ is chosen here.

are $S_{D^+D^-}^{\text{SM}} = -\sin(2\beta) = 0.69$, $S^{\text{HFAG}} = -0.75$ and $\Delta S = 0.52$ corresponds to two standard deviations. The values for the branching fraction are $\mathcal{B}_{D^+D^-}^{\text{SM}} = 1.7 \cdot 10^{-4}$, $\mathcal{B}_{D^+D^-}^{\text{HFAG}} = 1.9 \cdot 10^{-4}$ and $\Delta\mathcal{B} = 1.6$ corresponds to linear addition of the theoretical and experimental uncertainty.

A qualitative result that can be inferred from Fig. 3 is that the parameter space of a large positive CP asymmetry $C_{D^+D^-}$ is disfavoured by the bounds from the $S_{D^+D^-}$. This is easily seen from the formulae (42), (41) and the plots in appendix (A). A negative $C_{D^+D^-}$ demands a weak phase $\phi < 180^\circ$ and then the linear and quadratic terms in $S_{D^+D^-}$ add constructively and are in conflict with the consistent result between the SM and experiment in this observable. As for $B \rightarrow \tau\nu$ for small ρ_{DD} the linear terms dominate the quadratic ones and a regular pattern in $\cos(\phi)$ and $\sin(2\beta - \phi)$ emerges.

3.4 Discussion of $B_d \rightarrow D^+D^-$ and remarks on U-spin & colour related channels

A large CP asymmetry $C_{D^+D^-}$ would be a rather puzzling fact, as for instance discussed in Ref. [48]. One is lead to suspect that the gluonic penguin $B_d \rightarrow D\bar{q}q$ with $q = c$ might be enhanced by new physics. This scenario would or should lead to enhanced penguin amplitudes for $q = (u, d, s)$ as well and enter $B_d \rightarrow (\pi\pi, KK)$ in disagreement with the B -factory data.

We have seen that an unparticle scenario can lead, for appropriate parameters, to enhanced CP violation. One might wonder whether similar results shouldn't also show up in U-spin ($s \leftrightarrow d$) and colour related channels. The plots in Fig. 6 indicate that the CP asymmetry S in general does not necessarily receive large contributions. This can be inferred from Eq. (42) or by noting that the unparticles just contribute to a large SM background from $\sin(2\beta)$. We shall therefore focus on the CP asymmetry C . Let us note however that the situation for B_s decays is different since the mixing phase $\phi_s \simeq 0$ ($\phi_d \simeq 2\beta$) in the SM and the contributions of unparticles would be not be shielded by a large SM value.

The colour related or colour suppressed channel of $B_d \rightarrow D^+D^-$ is $B \rightarrow J/\Psi\pi_0$. The CP asymmetry has been measured $C_{J/\Psi\pi_0} = -0.11(20)$ [35], which is not conclusive in regard to its size. In the colour suppressed modes the non-factorizable contributions are enhanced due to different combinations of Wilson coefficients (typically $\sim 2-3$ larger than the factorizable amplitude) and have large strong phases. On the practical side it is harder to estimate them reliably in the SM and even more in the unparticle scenario, where the unparticle is dynamical as compared to the contracted W -boson propagator in the SM. The strong phases and the different hierarchy between factorizable and non-factorizable contributions in the SM and the unparticle scenario⁷ make it impossible to draw conclusions without explicit calculations.

⁷A parametric estimate gives that the non-factorizable contributions in the unparticle scenario are suppressed by a factor $2m_D^2/(m_{J/\Psi}^2 + m_B^2) \sim 0.2$ as compared to the SM.

The U-spin related transitions $b \rightarrow \bar{c}cs$ are CKM enhanced and therefore statistics should make them more attractive. In principle there is no reason that generic new physics respects the CKM hierarchy and U-spin. In the unparticle scenario there is no principle that dictates a CKM-like hierarchy in the coefficients $\lambda_{q'q}$ in the effective Lagrangian (10). Therefore they are not necessarily of major concern. Let us nevertheless discuss them. The gold plated decay $B \rightarrow J/\Psi K_s$ is also colour-suppressed. The measurement of the CP asymmetry $S_{J/\Psi K_s} = \sin(2\beta)$ has allowed determination of the angle β in the SM, whereas the CP asymmetry $C_{J/\Psi K_s} = 0$ is consistent with experiment. This mode is highly consistent with the SM or more precisely with one dominant amplitude. The branching fraction of the colour allowed decay $B_d \rightarrow D^+ D_s^-$ has been measured but no CP asymmetry has been reported, presumably because it does not exhibit CP violation in mixing. If the Belle CP asymmetry in $C_{D^+ D^-}$ gets confirmed a look at the CP asymmetry appears mandatory.

In summary the most interesting parallel channel is probably $B_d \rightarrow J/\Psi \pi^0$ and the improvement of the measurement in $C_{J/\Psi \pi^0}$ should be watched along with $C_{D^+ D^-}$. In the scenario we described we would generically expect a large CP asymmetry $C_{D^+ D^-}$ to be accompanied by a large asymmetry in $C_{J/\Psi \pi^0}$. It is a serious point of criticism, but on the other hand the experimental result is not conclusive and in theory there might be cancellations between the strong phase $e^{i d_u \pi}$ and the phase from the non-factorizable interactions. The time dependent CP asymmetries S are shielded by large SM backgrounds for B_d -meson, whereas in B_s system the SM expectation is $S \sim 0$ in many cases (e.g. $B_s \rightarrow J/\Psi \phi$) and the unparticle scenario might reveal itself.

We have seen that CP violation in $B_d \rightarrow D^+ D^-$ and $B \rightarrow \tau \nu$ can be maximal in the unparticle scenario. After this phenomenological section we shall elaborate on whether a CP asymmetry in leptonic decays is possible. Thereafter we shall turn to the question of whether the scale invariance at the TeV-scale or near scale-invariance could still be effective at heavy flavour scales $\sim 5 \text{ GeV}$.

4 Constraints from CPT on (new) CP-violation

The invariance under CPT symmetry imposes constraints on the amount of CP-violation; it enforces the equality of the partial sum of rates of particles and antiparticles⁸, to be made more precise below. Neither the SM nor any well-known new physics model predict CP-violation in leptonic decays such as $B \rightarrow \tau \nu$ studied in this paper. The aim of this section is to verify explicitly whether the CP-violation is consistent with the constraints from CPT.

Let us note that we expect that CPT-invariance holds for a theory with a local hermitian Lagrangian such as in Eq. (10). The explicit verification of CPT invariance demands that $\Theta \mathcal{L}(x) \Theta^{-1} = \mathcal{L}^\dagger(-x) = \mathcal{L}(-x)$, where $\Theta = CPT$ denotes the combined CPT-transformation. The Lagrangian (10) fulfills this requirement provided that

⁸I am grateful to Ikaros Bigi for drawing my attention to this fact.

$\Theta O_{\mathcal{U}}(x)\Theta^{-1} = O_{\mathcal{U}}^{\dagger}(-x)$, which we cannot verify explicitly since we do not have equations of motions or a Lagrangian for the unparticle field at hand from where we would infer the transformation under C , P and T . There also exists a general proof of the CPT-theorem in the framework of axiomatic field theory [51] based on general principles and axioms such as Lorentz invariance, uniqueness of the vacuum and causality of field commutators. Concerning the latter we would like to mention that we have seen in a previous section that the unparticle field obeys causality, c.f. Eq. (8). Summarising, although we are not able to prove CPT-invariance we at the same time do not find any indications why it should be violated.

It is well known that CPT symmetry implies equality of the decay rates of particles and antiparticles. In practice there is even a stronger consequence, e.g. [49], [33] or [50] where it was applied to charmless B -decays. The final state particles can be divided into subclasses of particles which rescatter into each other. It is a fact that the sum of the partial rates of these subclasses for a particle and its antiparticle must be the same. This can be inferred from the following relationship [49] between the weak decay amplitudes of a B -meson and its antiparticle \bar{B} to a final state f_x

$$\langle \bar{f}_x | H_{\text{decay}} | \bar{B} \rangle^* = \sum_i \langle f_x | S^{\dagger} | f_i \rangle \langle f_i | H_{\text{decay}} | B \rangle, \quad (45)$$

where H_{decay} corresponds to the weak transition operator and S is the scattering matrix. This relation is derived from the completeness relation $\mathbf{1} = \sum_i |f_i\rangle\langle f_i|$ and the fact that the CPT-operator is antiunitary. An equivalent but alternative relation on the level of decay rates can be found in Ref. [33]. From Eq. (45) it is then inferred that all states f_j which rescatter into f_x form a subclass whose partial rates of particles and antiparticles sum to zero

$$\sum_{i \in I} \Delta\Gamma(B \rightarrow f_i) = 0, \quad \langle f_i | S^{\dagger} | f_j \rangle \neq 0 \quad i, j \in I, \quad (46)$$

where

$$\Delta\Gamma(B \rightarrow f) \equiv \Gamma(B \rightarrow f) - \Gamma(\bar{B} \rightarrow \bar{f}) \quad . \quad (47)$$

The exact relation between the CP asymmetry and the difference of decay rates can be inferred from Eq. (17). Whereas the new CP asymmetry generated by $\mathcal{A}_{\text{CP}}(D^+D^-) \sim \Delta\Gamma(B_d \rightarrow D^+D^-)$ may be compensated by $\Delta\Gamma(B_d \rightarrow \bar{D}_0D_0)$ for instance, it is at first sight not clear which mode would compensate for the new CP asymmetry in $\mathcal{A}_{\text{CP}}(\tau\nu) \sim \Delta\Gamma(B^+ \rightarrow \tau^+\nu)$. Among the SM final states there does not seem to be an appropriate candidate. We are led to look in the unparticle sector for a suitable candidate. A firm hint can be gained by counting the coupling constants. Denoting the weak coupling by v and the unparticle coupling by λ (10), the CP asymmetry, which arises due to an interference of the two amplitudes depicted in Fig. 1, is of the order $O(\lambda^2 v^2)$. The processes $B^+ \rightarrow \mathcal{U}^+$ with an interference of the two amplitudes depicted in Fig. 7 has the same counting in the coupling constants. One amplitude corresponds to a tree decay and the other one

incorporates a virtual correction due to a fermion loop of the τ and the ν . The process $B^+ \rightarrow \mathcal{U}^+$ is kinematically allowed since the unparticle has a continuous mass spectrum. It does not proceed at resonance, but rather behaves like a multiparticle final state and is a realisation of Georgi's observation that the unparticle field in a final state behaves like a non-integral number $d_{\mathcal{U}}$ of massless particles.

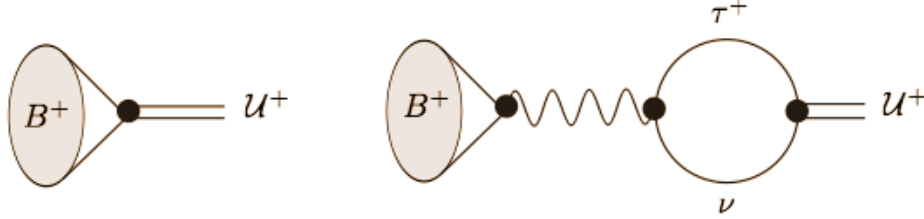


Figure 7: Decay $B^+ \rightarrow \mathcal{U}^+$, the double lines denote an unparticle (left) leading order (right) with virtual $\tau\nu$ -loop correction .

We shall now explicitly verify the CPT constraint

$$\Delta\Gamma(B^+ \rightarrow \tau^+\nu) + \Delta\Gamma(B^+ \rightarrow \mathcal{U}^+)_{\tau\nu\text{-loop}} = 0. \quad (48)$$

For the sake of simplicity we shall assume as previously that there is no flavour dependent perturbation in the neutrino sector and that $\lambda_P^{\tau\nu} = -\lambda_S^{\tau\nu}$ in (10). The formula for the first difference can be read off from Eq. (27)

$$\begin{aligned} \Delta\Gamma(B^+ \rightarrow \tau^+\nu) &= -4\mathcal{B}_{\tau\nu}^{\text{SM}} \sin(\phi) \sin(d_{\mathcal{U}}\pi) \Delta_{\tau\nu} \\ &= -\sin(\phi) \frac{G_F}{2\sqrt{2}\pi} \frac{m_B}{m_b} m_\tau f_B^2 \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 |\lambda_{\tau\nu}^S \lambda_{ub}^P V_{ub} U_{\tau\nu}| A_{d_{\mathcal{U}}} \left(\frac{m_B^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-1} \end{aligned} \quad (49)$$

Note that the cancellation of the phase factor $\sin(d_{\mathcal{U}}\pi)$ by the same factor in the denominator, as previously mentioned, is crucial for the cancellation here since the graphs in Fig. 7 do not involve this factor! The amplitude of the graph in Fig. 7 to the left is

$$\mathcal{A}(B^+ \rightarrow \mathcal{U}^+)_{\text{Fig.7(left)}} = \lambda_P^{\text{ub}*} \mathcal{A}_1 = \frac{\lambda_P^{\text{ub}*}}{\Lambda_{d_{\mathcal{U}}-1}} \frac{m_B^2}{m_b} f_B \langle P | O_{\mathcal{U}}^\dagger | 0 \rangle \quad (50)$$

and the amplitude of the graph to the right of Fig. 7 is

$$\begin{aligned} \mathcal{A}(B^+ \rightarrow \mathcal{U}^+)_{\text{Fig.7(right)}} &= \lambda_S^{\tau\nu*} V_{ub}^* U_{\tau\nu} \mathcal{A}_2 \\ &= \frac{\lambda_S^{\tau\nu*}}{\Lambda_{d_{\mathcal{U}}-1}} \frac{G_F}{\sqrt{2}} V_{ub}^* U_{\tau\nu} m_\tau f_B \Pi_{S-P}(m_B^2) \langle P | O_{\mathcal{U}}^\dagger | 0 \rangle \end{aligned} \quad (51)$$

where we have factored the weak parameters in $\mathcal{A}_{(1,2)}$. The fermion-loop Π_{S-P} is given by the correlation function

$$\Pi_{S-P}(p_B^2 = m_B^2) = i \int d^4x e^{-ip_B \cdot x} \langle 0 | T[\bar{\nu}(1-\gamma_5)\tau](x) [\bar{\tau}(1-\gamma_5)\nu](0) | 0 \rangle. \quad (52)$$

The decay rate is calculated from

$$\Gamma = \frac{|\mathcal{A}|^2}{2m_B} \int d\Phi \quad \text{with} \quad \int d\Phi = A_{d_{\mathcal{U}}}(m_B^2)^{d_{\mathcal{U}}-2} \quad (53)$$

being the phase space volume. The difference of decay rates is given by

$$\Delta\Gamma(B^+ \rightarrow \mathcal{U}^+)_{\tau\nu\text{-loop}} = 4 \sin(\phi) \text{Im}[\mathcal{A}_1^* \mathcal{A}_2] A_{d_{\mathcal{U}}}(m_B^2)^{d_{\mathcal{U}}-2} \quad (54)$$

Since \mathcal{A}_1 is real only the imaginary part of \mathcal{A}_2 will enter. The only strong phase is due to the τ and the ν going on-shell in the loop in Fig. 7 (right). Therefore we only need to know the imaginary part of the fermion loop which is given by

$$\text{Im}[\Pi_{S-P}(m_B^2 + i0)] = \frac{1}{4\pi} m_B^2 \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2. \quad (55)$$

Assembling the formulae we get

$$\Delta\Gamma(B^+ \rightarrow \mathcal{U}^+)_{\tau\nu} = \sin(\phi) \frac{G_F}{2\sqrt{2}\pi} \frac{m_B}{m_b} m_\tau f_B^2 \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 |\lambda_S^{\tau\nu} \lambda_P^{ub} V_{ub} U_{\tau\nu}| A_{d_{\mathcal{U}}} \left(\frac{m_B^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-1}, \quad (56)$$

which fulfills the CPT constraint Eq. (48) together with (49).

We have explicitly verified the CPT constraint (46) for the decay $B \rightarrow \tau\nu$ with unparticle-SM interactions given by the Lagrangian (10). We do not dare to speculate in any detail on how a decay $B^+ \rightarrow \mathcal{U}^+$ might be observed in a laboratory experiment. It can be said though that the unparticle has directed momentum, mass and charge which it directly inherits from the B -meson. Moreover in the case where there is a CP asymmetry in $B \rightarrow \tau\nu$ due to unparticles, it is precisely the CPT constraint (48) which tells us that there is an excess of charged unparticle degrees of freedom produced. Whether a part of this charge could annihilate into neutral particles or decay into charged particles remains unclear since the nature of this degree of freedom remains unknown at this stage. These questions could be addressed once a concrete model realising the unparticle scenario is known.

5 Breaking of scale invariance - dimensional analysis

The SM at the electroweak scale is not scale invariant. The logarithmic running and in particular the vacuum expectation value of the Higgs, which give masses to the fundamental particles, are responsible for the breaking of scale invariance. It is therefore a legitimate question at what scale the symmetry breaking will be transmitted to the unparticle sector by the effective Lagrangian (10). This will depend on the strength of the coupling and the relevance of the operators in the latter.

The authors of reference [5] have addressed this question, which we shall adapt accordingly for the weak sector. Assuming the unparticle field couples to an operator acquiring

a definite mass scale, the latter will break scale invariance at some energy. Let us assume for instance that the Higgs couples to the unparticle operator, following Ref. [5], as follows

$$\mathcal{L}^{\text{eff}} = \frac{\lambda_H}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}_0}-2}} |H|^2 O_{\mathcal{U}_0} \quad (57)$$

with $\lambda_H = c_{\mathcal{U}}^H (\Lambda_{\mathcal{U}}/M_{\mathcal{U}})^{d_{UV_0}-2}$ in our notation. We have used a new symbol $O_{\mathcal{U}_0}$ for the unparticle operator. This operator is not the same as the one used in Eq. (10) since it has to be electrically neutral. The important question for the analysis in this paragraph is what the value the anomalous dimension $\bar{d}_{\mathcal{U}}$ assumes. In the case where we think of the unparticle as being charged under $SU(2)_L$, $O_{\mathcal{U}_0}$ would appear as $\delta\mathcal{L}^{\text{eff}} \sim \bar{q}(\gamma_5)q O_{\mathcal{U}_0}$ in addition to the effective Lagrangian (10) and $d_{\mathcal{U}_0} = d_{\mathcal{U}}$ seems unavoidable. In the case where $O_{\mathcal{U}}$ is the only unparticle field then $O_{\mathcal{U}_0} = O_{\mathcal{U}} O_{\mathcal{U}}^\dagger$ would be a composite field with anomalous dimension in the range $0 \leq d_{\mathcal{U}_0} \leq 2d_{\mathcal{U}}$, where the Thirring model at coupling $\lambda = 2\pi$ [55] would be an example saturating the lower bound and supersymmetric QCD at the conformal IR fixpoint [52] an example saturating the upper bound. In the following we shall quote values for the bounds and the mean value explicitly. The Higgs VEV $\langle |H|^2 \rangle = v^2$ is expected to break scale invariance at a scale $\tilde{\Lambda}$

$$\frac{\lambda_H}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}_0}-2}} v^2 \tilde{\Lambda}^{d_{\mathcal{U}_0}} = \tilde{\Lambda}^4 \quad \Rightarrow \quad \tilde{\Lambda} = \Lambda_{\mathcal{U}} \left(\lambda_H \frac{v^2}{\Lambda_{\mathcal{U}}^2} \right)^{\frac{1}{4-d_{\mathcal{U}_0}}} . \quad (58)$$

What would this scale be in the cases we have investigated? Besides $\Lambda_{\mathcal{U}}$ there are two unknowns in the equation above, first $d_{\mathcal{U}}$ which appears explicitly in our results and $\lambda_H = c_{\mathcal{U}}^H (\Lambda_{\mathcal{U}}/M_{\mathcal{U}})^{d_{UV_0}-2}$. In the latter the matching coefficient will remain unknown but we can extract the ratio $(\Lambda_{\mathcal{U}}/M_{\mathcal{U}})$ from $\rho_{DD}(\rho_{\tau\nu})$ in terms of the UV dimensions. Taking $B \rightarrow DD$ as an example the breaking scale is

$$\tilde{\Lambda} = \Lambda_{\mathcal{U}} \left(c_{\mathcal{U}}^H \frac{v^2}{\Lambda_{\mathcal{U}}^2} \left(\frac{\rho_{DD}}{R_{DD}} \right)^{\frac{d_{UV_0}-2}{2(d_{UV}-1)}} \right)^{\frac{1}{4-d_{\mathcal{U}_0}}} , \quad (59)$$

where $R_{DD} = |c_S^{\text{cb}} c_S^{\text{cd}}|/|V_{\text{cb}} V_{\text{cd}}| = 1$, deviating it from 1 corresponds to a readjustment of ρ_{DD} in terms of the matching coefficients. Assuming for example $\Lambda_U = 1 \text{ TeV}$, $d_{\mathcal{U}} = 1.2$, $d_{\mathcal{U}_0} = (0, 1.2, 2.4)$, $\rho_{DD} = 10^{-3.5}$, the ratio of amplitudes and the breaking scale for fixed values of UV dimensions become

$$\begin{aligned} \Delta_{DD} &\simeq -0.40 R_{DD} \\ (d_{UV}, d_{UV_0}) &= (3, 6) \quad \tilde{\Lambda} \simeq (67, 20, 1) \text{ GeV} (R_{DD} c_{\mathcal{U}}^H)^{1/(4.0, 2.8, 1.6)} \\ (d_{UV}, d_{UV_0}) &= (3, 3) \quad \tilde{\Lambda} \simeq (300, 180, 50) \text{ GeV} (R_{DD}^{-1/4} c_{\mathcal{U}}^H)^{1/(4.0, 2.8, 1.6)} . \end{aligned} \quad (60)$$

The situation is not conclusive, which is not surprising bearing in mind that in the absence of a model there are simply too many unknowns. In the case where both UV dimensions are the same, which should be the case when $O_{\mathcal{U}}$ and $O_{\mathcal{U}_0}$ result from the same structure,

a small matching coefficient $c_{\mathcal{U}}^H$ is needed for a sizable effect at the heavy flavour scales. If the UV dimensions differ by a factor of two, which is the case when $O_{\mathcal{U}_0} = O_{\mathcal{U}} O_{\mathcal{U}}^\dagger$, effects are possible for moderate matching coefficient $c_{\mathcal{U}}^H$.

The effect $\Delta_{\text{DD}} = -0.40$ appears larger than the analysis or conclusions in Ref. [5] suggest. There are two reasons. First and simply, the CP violating phenomenon investigated in this paper is linear in the ratio of amplitudes, whereas [5] describes a case where the effect is proportional to the square of the amplitude. Secondly it was assumed that the SM Lagrangian has dimension four. The crucial point is that the weak Lagrangian has dimension six, $d_{\mathcal{L}_{\text{weak}}} = 6$ being suppressed by two powers of the weak scale, whereas the unparticle Lagrangian has dimension $d_{\mathcal{L}_{\text{unp}}} = d_{\mathcal{U}} + d_{\text{SM}}$. In terms of the effective Lagrangian (10) and the Eq. (9), $4 < d_{\mathcal{L}_{\text{unp}}} < 5$ the unparticle operator is more relevant than the weak operator. This gives rise to an enhancement factor in the amplitudes

$$\frac{(G_F/\sqrt{2})^{-1}}{\mu_{\text{HF}}^2} = \frac{8m_W^2}{g^2\mu_{\text{HF}}^2}, \quad (61)$$

which is explicit in the results of Eqs. (21), (41). In more physical terms one could state that the weak boson propagates at the high weak scale whereas the unparticle propagates at the low heavy flavour scale.

Adapting the analysis of Ref. [5] we imagine an experiment at a scale μ_{HF} , the unparticle Lagrangian (10) scales as $\mathcal{L}_{\text{eff}} = \lambda_S/\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}+(d_{\text{SM}}-4)}\mu_{\text{HF}}^{d_{\text{SM}}+d_{\mathcal{U}}}$, the weak Lagrangian as $\mathcal{L}_{\text{weak}} \simeq G_F\mu_{\text{HF}}^6$ and the ratio is

$$\Delta \simeq \lambda_{\mathcal{U}}^S \left(\frac{\Lambda_{\mathcal{U}}}{M_{\mathcal{U}}}\right)^{d_{UV}-d_{\mathcal{U}}} \left(\frac{\mu_{\text{HF}}}{M_{\mathcal{U}}}\right)^{d_{\mathcal{U}}+d_{\text{SM}}-6} \left(\frac{G_F^{-1}}{\mu_{\text{HF}}^2}\right). \quad (62)$$

Imposing that the energy scale of the experiment is higher than the breaking scale, i.e. $\mu_{\text{HF}} > \tilde{\Lambda}$, the following bound is obtained⁹

$$\Delta < \frac{c_{\mathcal{U}}^S}{c_{\mathcal{U}}^H} \left(\frac{\mu_{\text{HF}}}{M_{\mathcal{U}}}\right)^{d_{\text{SM}}-2} \left(\frac{\mu_{\text{HF}}^2}{v^2}\right) \left(\frac{G_F^{-1}}{\mu_{\text{HF}}^2}\right) \left(\frac{\mu_{\text{HF}}}{\Lambda_{\mathcal{U}}}\right)^{d_{\mathcal{U}}-d_{\mathcal{U}_0}}. \quad (63)$$

This equation is easily interpreted. The first factor measures the ratio of the two couplings. The second is a measure between the relevance or dimension of the SM operator that is coupled to the unparticle and the dimension of the Higgs operator. In the third term the scale of the experiment has to compete with the Higgs VEV. The fourth term is peculiar to the weak interactions, as described above, and is due to the fact that the weak process takes place at the weak scale G_F^{-1} and the unparticle propagates at the low scale μ_F . The fifth term is due to the difference of anomalous dimensions of the charged unparticle operator in the effective Lagrangian (10) and the neutral unparticle operator coupling to the Higgs VEV (57), whether it acts as an enhancing or decreasing factor depends on the anomalous dimensions. In a later paper [6] further dimensional analysis is explored. It

⁹Setting $c_{\mathcal{U}} \rightarrow 1$, the fourth and the fifth term to one and taking the square root of the equation, the bound in Ref. [5] is recovered with $\Delta^2 = \epsilon$.

is observed that when the coupling λ multiplied by the suppression scale $1/\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}+(d_{\text{SM}}-4)}$ is combined into a single scale $1/\Lambda_{(d_{\text{SM}})}^{d_{\mathcal{U}}+(d_{\text{SM}}-4)}$ then under the assumption $1 < d_{d_{\mathcal{U}}} < 2 < d_{UV}$, it is inferred that $\Lambda_2 < M_{\mathcal{U}} < \dots < \Lambda_4 < \Lambda_3$ which seems counterintuitive at first sight since higher dimensional operators could receive an enhancement.

Dimensional analysis is not very reliable. The construction of an explicit model would help to answer questions and presumably constrain the structure of the effective Lagrangian (10).

Possible candidates are extensions of the standard model featuring near conformal dynamics, such as the walking technicolor theories. Those theories are close to an infrared fixpoint and hence have slow varying coupling constants. A complete extension of the SM featuring walking dynamics and its link to the underlying gauge theory has been given in Ref. [31].

6 Critical discussion and conclusions

In this paper we have investigated the consequences of the unparticle scenario in heavy flavour physics. The new feature is a CP odd or strong phase that arises in the propagator as a consequence of the non integral scaling dimension. This gives rise to very characteristic and novel CP violating phenomena.

The drawbacks of the scenario are that there is as yet no concrete model and that it is not clear to what energies the scale invariant sector extends. The lack of a model is overcome by parametrizing an effective Lagrangian, c.f. (10), at the cost of many unknown coefficients which have to be constrained. We have investigated the extension of the scale invariant sector to lower energies resorting to dimensional analysis. We have found that effects at the heavy flavour scales are possible provided the coupling of the unparticle field to the Higgs VEV is moderate at the scale $\Lambda_{\mathcal{U}}$. The effects are sizable for two reasons. Firstly the scaling dimension of the unparticle Lagrangian is more relevant than the one of the weak Lagrangian and secondly the effect of CP violation is linear and not shielded by a large SM background.

Bearing in mind the breaking of scale invariance we have chosen decays where the unparticle propagates at a relatively large scale. The two examples we have investigated are the decays $B^+ \rightarrow \tau^+ \nu$ and $B_d \rightarrow D^+ D^-$. In doing so we have assumed the scale invariant sector extends to the scale $\sim 5 \text{ GeV}$ for the former and to $\sim 2 \text{ GeV}$ for the latter. We have not considered decays into final state particles as in for instance Ref. [1] They would also lead to signals but we have assumed the unparticles to be weakly coupled.

We have chosen cases where the SM is described by a single weak amplitude and the unparticles add a second weak amplitude with strong phase allowing for the CP violation. In this sense our analysis does not differ from other model analyses with two amplitudes. The particularity of the unparticle scenario as compared to other models is that it is an example where the large strong phase might be generated by the strength of the coupling constant and that the contribution to other (flavour)-channels is qualitatively

different from other models, for example from those generating the strong phases through penguins.

The prediction of a CP asymmetry $\mathcal{A}_{CP} = -C$ for leptonic decays seems a unique feature of the unparticle model, which has puzzling consequences to be discussed below. The reparametrization invariant is the product of two quadratic invariants, one from the quark sector and one from the lepton sector. As an example we have looked at $B^+ \rightarrow \tau^+ \nu$ in conjunction with the constraints from the branching ratio. Generic plots for the parameter space of the anomalous dimension and the weak phase difference are shown in Fig. 3. Maximal CP violation is possible for certain values of the parameter space. The current experimental data is not yet strong enough to set absolute bounds. Comments on flavour related decays are given in 3.2. In particular the channel $D \rightarrow \mu \nu$ might be of interest since more events have been collected [39] than in $B \rightarrow \tau \nu$ [37], [38]. To the knowledge of the author there are no experimental data available with bounds on CP asymmetries in leptonic decays. Charge symmetry is usually implied in the analysis.

The investigation of the non-leptonic decay $B_d \rightarrow D^+ D^-$ was motivated by the large asymmetry $C_{D^+ D^-}$ reported by Belle [43]. We have neglected the penguin contribution and treated the decay in naive factorization. As compared to $B \rightarrow \tau \nu$ there is a third observable, the time dependent CP asymmetry $S_{D^+ D^-}$. The latter agrees rather well with the SM predictions and sets constraints on $C_{D^+ D^-}$. It is possible though to find values where the CP violation is maximal and satisfies the constraints of the branching ratio and the time dependent CP asymmetry. As for $B \rightarrow \tau \nu$, plots for generic parameters are shown in Fig. 6. It is encouraging that for small ratios of effective couplings the constraints from $S_{D^+ D^-}$ allow for a large negative asymmetry $C_{D^+ D^-}$ as reported by Belle whereas the opposite sign seems to be disfavoured. This fact is general to any analysis with two amplitudes as outlined in section 2.2; the unparticles just provide a scenario with two amplitudes and possible large weak and strong phase differences. The true meaning is that in the case where the decay is described by two amplitudes, the sign of the Belle measurement is more consistent than the opposite sign. Discussions on U-spin and colour related decays are given in section 3.4. Let us emphasize two points from this section once more. Generically we would expect a large asymmetry in $C_{D^+ D^-}$ to be accompanied by a large asymmetry in the color related $C_{J/\Psi \pi_0}$. Currently the experimental value $C_{J/\Psi \pi_0}^{\text{PDG}} = -0.11(20)$ [35] is not conclusive and moreover $B_d \rightarrow J/\Psi \pi_0$ and on the theoretical side, complications arise due to non-factorizable contributions. For B_d decays the time dependent asymmetries are typically proportional to $\sin(2\beta)$ or $\sin(2\alpha)$, the large angles of the B_d triangle, and new physics contributions are therefore hard to see. For B_s decays, the mixing phase is $\phi_s \simeq 0$ and therefore the unparticle scenario could give rise to sizable corrections. This would be particularly interesting for $B_s \rightarrow J/\Psi \phi$ which aims at the extraction of the B_s mixing phase ϕ_s at the LHCb.

We have verified in section 4 that the novel CP-violation satisfies constraints from CPT-invariance, namely the equality of the sum of partial rates, of the subclasses of final states rescattering into each other, of particle and antiparticle. Since the SM and no well-known new physics model predicts a CP asymmetry for leptonic decays such as

$B^+ \rightarrow \tau^+ \nu$ we have inferred that the compensating mode must be due to unparticles. As we have quantitatively verified, the compensating mode is $B^+ \rightarrow \mathcal{U}^+$. This might appear surprising at first sight but is possible since the unparticle does not have a definite mass but a continuous spectrum like a multiparticle state which was one of the basic observations in Georgi's first paper [1].

Clearly the unparticle scenario would benefit largely from the construction of an explicit model. The question of the breaking of scale invariance and what a real¹⁰ unparticle in a laboratory experiment would mean could be addressed and it would presumably also provide structural constraints on the coefficients of the effective Lagrangian.

Acknowledgments

I am grateful to Ikaros Bigi, Oliver Brein, Luigi Del Debbio, Sakis Dedes, Stefan Förste, Uli Haisch, Jörg Jeckel, Francesco Sannino, Christopher Smith, Raymond Stora for discussions, to Nikolai Uraltsev for correspondence on the $B \rightarrow D$ form factor, to Paul Jackson, Sheldon Stone, Erika de Lucia and Roberto Versaci for correspondence, to Tom Underwood for help with figures and to Lara Mary Turner for reading of the manuscript. Comments are welcome.

This work was supported in part by the EU networks contract Nos. MRTN-CT-2006-035482, FLAVIANET, and MRTN-CT-2006-035505, HEPTOOLS.

A Some plots as a function of $d_{\mathcal{U}}$

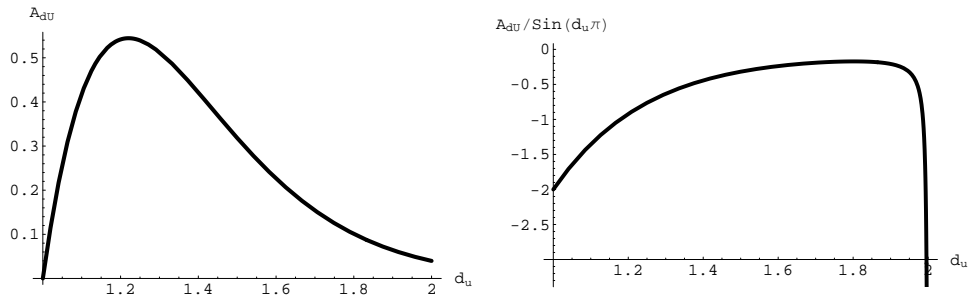


Figure 8: The phase space function $A_{d_{\mathcal{U}}}$ (5) (left) and as appearing in the propagator $A_{d_{\mathcal{U}}} / \sin(d_{\mathcal{U}} \pi)$ (6) plotted against $d_{\mathcal{U}}$

¹⁰As opposed to a virtual particle, on which we focused throughout this paper.

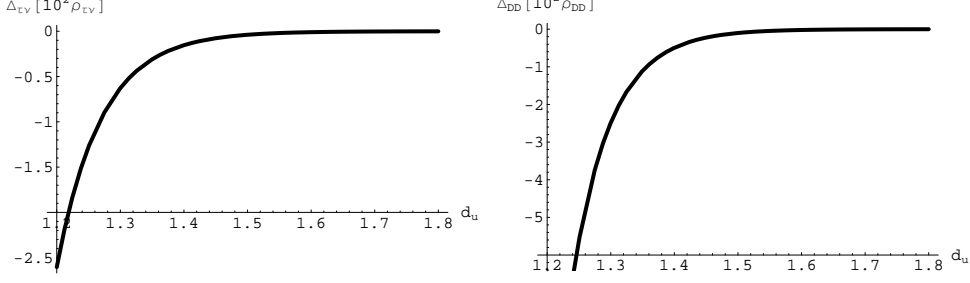


Figure 9: The ratio of unparticle to SM model amplitude $\Delta_{\tau\nu(DD)}$ in units of the ratio of effective couplings $[\rho_{\tau\nu(DD)} \cdot 10^2]$ (30)(44), versus the anomalous dimension d_u . (left) $\Delta_{\tau\nu}$ (30), (right) Δ_{DD} (44)

B Explicit results in coordinate space

B.1 The commuator

The commutator of the unparticle field

$$C(x) = \langle 0 | [O_U(x), O_U(0)] | 0 \rangle \quad (\text{A.1})$$

may be obtained from the time ordered product

$$C_F(x) = \langle 0 | T O_U(x) O_U(0) | 0 \rangle \quad (\text{A.2})$$

via the general formula

$$C(x) = -2i \text{sgn}(x_0) \text{Im}[C_F(x)] \quad . \quad (\text{A.3})$$

The correlation function $C_F(x)$ is obtained by Fourier transformation of (6), c.f. Ref. [53]

$$C_F(x) = \int \frac{d^4 P}{(2\pi)^4} e^{-iPx} (-i\Delta_U(P^2)) = -\frac{2^{d_U-4}}{\pi^2} \frac{A_{d_U}}{2 \sin(d_U \pi)} \frac{\Gamma(d_U)}{\Gamma(2-d_U)} (-x^2 + i0)^{-d_U} . \quad (\text{A.4})$$

The imaginary part is

$$\text{Im}[(-x^2 + i0)^{-d_U}] = -\sin(d_U \pi) \Theta(x^2) (x^2)^{-d_U} \quad (\text{A.5})$$

and we obtain the commutator

$$\begin{aligned} C(x) &= -i \text{sgn}(x_0) \Theta(x^2) (x^2)^{-d_U} \sin(d_U \pi) \frac{2^{2d_U-4}}{\pi^2} \frac{A_{d_U}}{\sin(d_U \pi)} \frac{\Gamma(d_U)}{\Gamma(2-d_U)} \\ &= -i \text{sgn}(x_0) \Theta(x^2) (x^2)^{-d_U} (d_U - 1) \frac{\Gamma(d_U + 1/2) \pi^{1/2-2d_U}}{\Gamma(2d_U) \Gamma(2-d_U)} \end{aligned} \quad (\text{A.6})$$

The free field case $d_U \rightarrow 1$

$$\lim_{d_U \rightarrow 1+} C(x) = \frac{-i}{2\pi} \text{sgn}(x_0) \delta(x^2) \quad (\text{A.7})$$

may be recovered by use of the formula $\lim_{\epsilon \rightarrow 0+} \epsilon |z|^{1-\epsilon} = \delta(z)$. Or for any integer n

$$\lim_{d_U \rightarrow n+} C(x) \sim -i \text{sgn}(x_0) \delta^{(n-1)}(x^2) ,$$

it is seen that the commutator has support on the light-cone only.

B.2 The Thirring model - an example with phase factor

The Thirring model belongs to the class of exactly solvable two dimensional models. It is a fermionic model with a vector current-current interaction. The exact solution of the two point function was obtained by Johnson [54] as a function of free fields

$$\langle 0|T\Psi(x)\bar{\Psi}(0)|0\rangle = -ie^{-i4\pi\gamma D_0(x)}G_0(x), \quad (\text{A.8})$$

where

$$D_0(x) = \frac{-i}{4\pi} \log(-x^2 + i0) \quad G_0(x) = \frac{1}{2\pi} \frac{\gamma_\mu x^\mu}{x^2 - i0} \quad (\text{A.9})$$

are the free bosonic and fermionic Greens functions. We have identified $\gamma = (\lambda^2/4\pi^2)(1 - \lambda^2/4\pi^2)^{-1}$, where λ is the current-current coupling constant, as for instance in [55]. N.B. $\gamma > 0$ in accordance with (9) and $d_{\mathcal{U}} = 1 + \gamma$. We recover

$$\langle 0|T\Psi(x)\bar{\Psi}(0)|0\rangle = \frac{i}{2\pi} \frac{\gamma_\mu x^\mu}{(-x^2 + i0)^{1+\gamma}} \quad (\text{A.10})$$

the formula (A.4) in the fermionic case up to an overall normalization. The phase factor arises due to resummation of thresholds at $x^2 > 0$. Note that the overall normalization in a scale invariant theory is a matter of convention and is hidden in the arbitrary scale factor in the logarithm of the free bosonic function $\log[(-x^2 + i0)\mu^2]$. In the notation of Eq. 2 the scale μ is proportional to the fixed point scale $\Lambda_{\mathcal{U}}$. This scales exhibits the phenomenon of dimensional transmutation.

C Final state interaction in $B_d \rightarrow D^+ D^-$ consistent with naive factorization

In this appendix we shall obtain the isospin final state interaction phases within the naive factorization approach. The isospin analysis from $K \rightarrow \pi\pi$ and $B \rightarrow \pi\pi$, c.f. [33] is transferable to $\bar{B}_d \rightarrow D^+ D^-$ [44]. The D -mesons are $I = 1/2$ states. Angular momentum conservation implies that only $I = 0$ and $I = 1$ states are formed as final states in the decay. Denoting the amplitudes

$$A^{+-} = \mathcal{A}(\bar{B}_d \rightarrow D^+ D^-) \quad A^{+0} = \mathcal{A}(\bar{B}_d \rightarrow D^0 D^0) \quad A^{+0} = \mathcal{A}(\bar{B}_u \rightarrow D^+ D^0),$$

isospin symmetry implies

$$A^{+0} = A_1 \quad A^{+-} = \frac{1}{2}(A_1 + A_0) \quad A^{00} = \frac{1}{2}(A_1 - A_0) \quad (\text{A.11})$$

from where the famous isospin triangle follows

$$A^{+0} = A^{+-} + A^{00} \quad . \quad (\text{A.12})$$

Let us introduce the following notation

$$A_0 = Z_0 e^{i\delta_0} \quad A_1 = Z_1 e^{i\delta_1} \quad , \quad (\text{A.13})$$

where we have factorized the final state interaction phase in the corresponding isospin channels.

In the naive factorization approximation $Z_0 = Z_1 \equiv Z$ and therefore

$$\begin{aligned} A^{+0} &= Z \cos\left(\frac{\delta_1 - \delta_0}{2}\right) e^{i(\delta_1 - \delta_0)/2} \\ A^{00} &= iZ \sin\left(\frac{\delta_1 - \delta_0}{2}\right) e^{i(\delta_1 - \delta_0)/2} \\ A^{+0} &= Z e^{i\delta_1} . \end{aligned} \quad (\text{A.14})$$

The isospin triangle becomes rectangular

$$|A^{+0}|^2 = |A^{+-}|^2 + |A^{00}|^2 \quad (\text{A.15})$$

Two out of the three rates have been measured [35]

$$\bar{B}(B_d \rightarrow D^+ D^-) = 1.9(6) \cdot 10^{-4} \quad \bar{B}(B^+ \rightarrow \bar{D}^0 D^+) = 4.8(1) \cdot 10^{-4} . \quad (\text{A.16})$$

Neglecting irrelevant phase space effects we obtain from (A.14)

$$\cos^2\left(\frac{\delta_1 - \delta_0}{2}\right) \simeq 0.4(2) . \quad (\text{A.17})$$

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